ROBUST GUARANTEED COST CONTROL FOR DISCRETE-TIME SYSTEMS WITH MULTIPLE DELAYS IN STATE

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Abstract. This paper deals with the problem of robust guaranteed cost control for generalized discrete time-delay systems with norm-bounded uncertainties. Based on linear matrix inequality (LMI) technique, we develop discrete Lyapunov function approach for robust performance analysis and robust stabilization via linear memoryless state feedback. We show that the feasibility of a linear matrix inequality guarantees the solvability of the addressed robust guaranteed cost control problem. Moreover, we show that the robust guaranteed cost control for the generalized discrete time-delay systems with norm-bound uncertainties can be viewed as an $H_\infty$ type condition for an uncertainty-free system. We also have presented an upper bound for the cost function of the underlying system with the designed controller for the case of initial state values being a zero mean random variables. An example is given to show the potential of the proposed techniques.

Keywords. Discrete-time system, Guaranteed cost control, Robust control, Multiple time-delay, parametric uncertainty.

AMS (MOS) subject classification: 37N35, 93C55

1 Introduction

Robust stability and stabilization of dynamic systems which include time delays and uncertainties in their physical models are problems of recurring interest because the existence of delays and uncertainties often induce instability (see for example, [6, 11, 9]). The interest is justified by the fact that stability is the most important objective in control system design.

In recent years, with the development of robust control and $H_\infty$ control theory, robust guaranteed cost control approach to the design of state feedback control laws for uncertain systems has been a subject of intensive research: see, e.g. quadratic guaranteed cost control for uncertain non-delay systems was dealt with in the work of, for example, [12, 4, 17, 28], and the continuous counterpart of the above problem was considered in, for example, [10, 13, 7]. On the other hand, the study on control and filtering of discrete-time uncertain systems with delays has quite active for the last decade, and a number of results have been reported in the literature, for
example, [27, 14, 25, 26]. The problem of robust guaranteed cost control for discrete-time uncertain systems with delay is considered in [5] for deterministic systems, and in [1] for stochastic systems with markovian jumps. Note that, however, the results obtained in [5] are concerned with single time-delay only in state, which may be conservative in some real systems, since multiple-delays occurs quite often in system plants. In this paper, we developed further the method adopted in [5] to solve the problem of robust control for discrete time systems with both multiple time-delays and parametric uncertainties. Robust guaranteed cost control for this type of system via memoryless state feedback is designed such that the resulting closed-loop system is stable for all admissible uncertainties and an adequate level of performance is guaranteed. The proposed methods are given in terms of a linear matrix inequality (LMI). It is shown that the feasibility of a linear matrix inequality (LMI) ensures the existence of memoryless state feedback controller which solves the addressed problem. Once the feasibility of LMI is determined, a family of desired memoryless state feedback control law is then can be constructed. The advantage of LMI approach is that the problem of finding the optimal cost can be easily solved without requiring the tuning of any parameters. The LMI approach has also the advantage that it can be solved numerically very efficiently by using interior-point algorithms which have been developed recently (see [3]). Moreover, it is demonstrated that the above robust guaranteed cost control problem can be regarded as an $H_{\infty}$ type condition for an uncertainty-free system. Finally, an upper bound for the cost function is provided.

The motivation for this research stems from three fold: (i) guaranteed cost control has many important applications in engineering and science, and many real systems are modelled as discrete-time systems with unknown time-delays; (ii) in practical, it is almost always impossible to get an exact mathematical model of a dynamical system due to the complexity of the system, the difficulty of measuring various parameters, environmental noises, uncertain and/or time-varying parameters, etc. Indeed, the model of the system to be controlled almost always contains some type of uncertainty. Thus, robustness requirement (robust stability of the uncertain closed-loop system and the robust performance) is essential and important in control system design; and (iii) although the signals of interest in control systems are usually continuous and the performance specification is formulated in continuous time, some continuous-time signals are sampled at certain time instants in systems operating. One of the common approaches to design a digital controller for a continuous-time systems is to obtain a discrete-time model of the system, by discretizing a continuous-time model, and then design a discrete-time controller.

The paper is organized as follows: Section 2 is devoted to the problem of robust performance analysis. The memoryless state feedback control design is investigated in Section 3, and an illustrative example is given in Section 4.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote, respectively,
the $n$ dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript “$T$” denotes matrix transposition and the notation $X \geq Y$ (respectively, $X > Y$) where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $I$ is the identity matrix. The symbol $\lambda_{\text{max}}(A)$ denotes the maximal eigenvalues of a square matrix $A$, while $\sigma_{\text{max}}(A)$ is the maximal singular value of matrix $A$, i.e., $\sigma_{\text{max}}(A) = \sqrt{\lambda_{\text{max}}(A^T A)}$.

2 Robust Performance Analysis

Consider the following discrete-time system with multiple time delays and parameter uncertainties described by

$$
\begin{align*}
\left\{ \begin{array}{l}
    x(k+1) = (A_0 + \Delta A_0)x(k) + \sum_{i=1}^{n} (A_i + \Delta A_i)x(k - d_i), \\
    x(k) = \phi(k), \ k \in \{-d, -d + 1, \cdots, 0\}, \ d = \max\{d_1, d_2, \cdots, d_n\},
\end{array} \right.
\end{align*}
$$

where $x(k) \in \mathbb{R}^n$ is the state, $A_0$ and $A_i (i = 1, 2, \ldots, n)$ are known real constant matrices of appropriate dimensions which describe the nominal system, $\Delta A_0, \Delta A_i (i = 1, 2, \ldots, n)$ are real-valued matrix functions representing the time-varying parameter uncertainties, $d_i (i = 1, 2, \ldots, n)$ is positive integer for time delays and $\phi(k)$ is an initial state value at time $k$. The admissible parameter uncertainties considered here are of the form:

$$
\begin{bmatrix}
\Delta A_0 & \Delta A_1 & \cdots & \Delta A_n
\end{bmatrix} = HF(k) \begin{bmatrix}
E_0 & E_1 & \cdots & E_n
\end{bmatrix},
$$

where $H, E_i, (i = 0, 1, \cdots, n)$ are known real constant matrices of appropriate dimensions which capture the structure of the uncertainties. $F(k) \in \Omega$ is the uncertainty source, where $\Omega$ is defined as

$$
\Omega = \left\{ F(k) : \sigma_{\text{max}}(F(k)) \leq 1, \quad k \geq 0 \right\}.
$$

Associated with system (1) is the following quadratic cost function:

$$
J = \sum_{k=0}^{\infty} x^T(k)Qx(k), \quad Q = Q^T > 0.
$$

For the sake of technical simplification, without loss of generality, the matrix $Q$ would be factored as $Q = D^T D$.

Remark 2.1 It is worthwhile to pointing out that the uncertainty structure of (1) satisfying (2)-(3) has been widely used in robust control and filtering for uncertain systems for both deterministic and stochastic cases, see, for example, [2, 1, 8, 15, 16, 18, 23, 19, 22, 21, 24, 20] and the references therein.
The matrix $F(k)$ is allowed to be state-dependent, i.e., $F(t) = F(k, x(k))$, as long as (3) is satisfied along all possible state trajectories. Note that the matrix $F(k)$ contains the uncertain parameters in the state and input matrices of system (1). The matrices $H, E_0, E_1...$ and $E_n$ specify how the uncertain parameters in $F(t)$ affect the nominal matrices of system (1). Also, observe that the unit overbound for $F(k)$ in (3) does not cause any loss of generality. Indeed, $F(k)$ can be always normalized, in the sense of (3), by appropriately choosing the matrices $H, E_0, E_1...$ and $E_n$.

We recall the following Lemmas which will be needed in the proof of our main results.

**Lemma 2.1** (Schur Complement) Given constant matrices $\Omega_1, \Omega_2, \Omega_3$ where $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$ then

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$$

if and only if

$$\begin{bmatrix}
\Omega_1 & \Omega_3^T \\
\Omega_3 & -\Omega_2
\end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix}
-\Omega_2 & \Omega_3 \\
\Omega_3^T & \Omega_1
\end{bmatrix} < 0.$$

**Lemma 2.2** ([29]) Given a symmetric matrix $Y \in \mathbb{R}^{n \times n}$, and two real matrices $H, E$ with appropriate dimensions. The following matrix inequality holds for all $F : F^T F < I$, i.e.

$$Y + HFE + E^T F^T H^T < 0,$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$Y + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0.$$

The robust performance analysis problem associated with the uncertain system (1) is stated as follows:

Determine if system (1) is uniformly asymptotically stable and find an upper bound for the cost function (4) for all admissible uncertainties satisfying (2)-(3).

We now have the following result to solve the robust performance analysis problem.

**Theorem 2.1** Consider system (1) with uncertainties satisfying (2)-(3). Then, the robust performance analysis problem is solvable if there exist symmetric matrices $X > 0, R_i > 0$ ($i = 1, 2, \ldots, n$) and a positive scalar $\varepsilon$ such that the following LMI is feasible:

$$X - \varepsilon HH^T > 0, \quad \text{and} \quad X - \sum_{i=1}^{n} R_i > 0$$
\[
\begin{bmatrix}
X - \varepsilon HH^T & A_0X & A_{dx} & 0 & 0 \\
XA_0^T & X - \sum_{i=1}^n R_i & 0 & XD^T & XE_0^T \\
A_{dx}^T & 0 & R & 0 & E_{dx}^T \\
0 & DX & 0 & I & 0 \\
0 & E_0X & E_{dx} & 0 & \varepsilon I
\end{bmatrix} > 0, \quad (5)
\]

where

\[
A_{dx} = \begin{bmatrix} A_1X & A_2X & \cdots & A_nX \end{bmatrix}, \quad E_{dx} = \begin{bmatrix} E_1X & E_2X & \cdots & E_nX \end{bmatrix}
\]

Moreover, the cost function (4) satisfying the following bound:

\[
J(k) \leq x^T(0)X^{-1}x(0) + \sum_{i=1}^n \sum_{j=-d_i}^{-1} x^T(j)X^{-1}R_iX^{-1}x(j). \quad (6)
\]

**Proof.** In the following, for simplicity, we will define

\[
\tilde{A}_0 = A_0 + \Delta A_0, \quad \tilde{A}_i = A_i + \Delta A_i(i = 1, 2, ..., n).
\]

Let the following discrete Lyapunov function candidates for the system (1):

\[
V(x(k)) = x^T(k)P x(k) + \sum_{i=1}^n \sum_{j=-d_i}^{-1} x^T(k+j) \tilde{R}_i x(k+j), \quad (7)
\]

where \( \tilde{R}_i = PR_iP \), while \( P \) and \( R_i (i = 1, 2, ..., n) \) are symmetric positive-definite matrices.

Taking the forward difference along with the system (1), we obtain:

\[
\Delta V(x(k)) = x^T(k)Px(k) + \sum_{i=1}^n (x^T(k) \tilde{R}_i x(k) - x^T(k-d_i) \tilde{R}_i x(k-d_i))
\]

\[
= \begin{bmatrix}
\frac{\tilde{A}_0}{\tilde{A}_1^T} & \frac{\tilde{A}_1}{\tilde{A}_2^T} & \cdots & \frac{\tilde{A}_n}{\tilde{A}_n^T}
\end{bmatrix}
\begin{bmatrix}
P \\
P \\
P \\
P
\end{bmatrix}
\begin{bmatrix}
\frac{\tilde{A}_0}{\tilde{A}_1^T} & \frac{\tilde{A}_1}{\tilde{A}_2^T} & \cdots & \frac{\tilde{A}_n}{\tilde{A}_n^T}
\end{bmatrix}
\begin{bmatrix}
x(k) \\
x(k) \\
x(k) \\
x(k)
\end{bmatrix}
\]

\[
- \begin{bmatrix}
x(k) \\
x(k-d_1) \\
\vdots \\
x(k-d_n)
\end{bmatrix}
\begin{bmatrix}
P - \sum_{i=1}^n \tilde{R}_i & 0 & 0 & 0 \\
0 & R_1 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \tilde{R}_n
\end{bmatrix}
\begin{bmatrix}
x(k) \\
x(k-d_1) \\
\vdots \\
x(k-d_n)
\end{bmatrix}.
\]
Note that if the following inequality
\[
\begin{bmatrix}
P^{-1} & \tilde{A}_0 & \tilde{A}_d \\
\tilde{A}_0^T & P - \sum_{i=1}^n R_i - Q & 0 \\
\tilde{A}_d^T & 0 & R
\end{bmatrix} > 0,
\]
is true, where
\[
\tilde{A}_d = \begin{bmatrix} 
\tilde{A}_1 & \cdots & \tilde{A}_n
\end{bmatrix}, \quad R = \text{diag}(R_1, \cdots, R_n),
\]
from Lemma 2.1, we can have the conclusion that
\[
\Delta V (x(k)) < -x^T(k) Q x(k) < 0.
\]
Noting \(Q > 0\), hence, it follows from Lyapunov stability theory that inequality (9) implies system (1) is globally uniformly asymptotically stable.

By pre-multiplying and post-multiplying the matrix \(\text{diag}(I; P^{-1}; P^{-1})\) to the matrix inequality (8), and let \(X = P^{-1}\), one has
\[
\begin{bmatrix}
X & A_0 X \\
X A_0^T & X - \sum_{i=1}^n R_i - X Q X \\
A_0 X & 0 \\
\tilde{A}_d x & R
\end{bmatrix} > 0,
\]
where \(\tilde{A}_d x = \begin{bmatrix} \tilde{A}_1 X & \tilde{A}_2 X & \cdots & \tilde{A}_n X \end{bmatrix}\), \(R = \text{diag}(R_1, R_2, \cdots, R_n)\).

We define
\[
\Xi = \begin{bmatrix}
X & A_0 X \\
X A_0^T & X - \sum_{i=1}^n R_i - X Q X \\
A_0 X & 0 \\
A_{dx} & R
\end{bmatrix},
\]
where \(A_{dx} = \begin{bmatrix} A_1 X & A_2 X & \cdots & A_n X \end{bmatrix}\). Then the matrix inequality (10) is equivalent to the following one:
\[
\Xi + 2 \begin{bmatrix}
H \\
0 \\
0
\end{bmatrix} F(k) \begin{bmatrix} 0 & E_0 X \\
E_0 X & E_{dx} \end{bmatrix} > 0,
\]
where \(E_{dx} = \begin{bmatrix} E_1 X & E_2 X & \cdots & E_n X \end{bmatrix}\). In the light of Lemma 2.2, the above inequality is true for all admissible uncertainties if and only if there exists a positive scalar \(\varepsilon\) such that
\[
\Xi - \begin{bmatrix}
\varepsilon H H^T & 0 \\
0 & \varepsilon^{-1} X E_0^T E_0 X & 0 \\
0 & 0 & \varepsilon^{-1} E_{dx}^T E_{dx} X
\end{bmatrix} > 0.
\]

Finally, we know that the matrix \(Q\) can be factored as \(Q = D^T D\), so we obtain that inequality (11) is equivalent to the linear matrix inequality (5) in Theorem 2.1 from lemma 2.1.
Next, we will show the cost function (4) satisfies the upper bound in (6). It follows from inequality (9) that

\[ x^T(k)Qx(k) \leq -\Delta V(x(k)). \]

Summing both sides of the above inequality from \( k = 0 \) to \( \infty \) leads to

\[ \sum_{k=0}^{\infty} x^T(k)Qx(k) \leq V(x(0)) - V(x(\infty)). \]

Since the quadratic stability of the system has already been established, we conclude that \( V(x(k)) \to 0 \) as \( k \to \infty \). Hence we can obtain

\[ J(k) \leq x^T(0)Px(0) + \sum_{i=1}^{n} \sum_{j=-d_i}^{-1} x^T(j)PR_iPx(j), \]

which is the end of the proof.

Notice that the LMIs (5) and (6) have the same structure as those results in [5], where there is only single time-delay in state.

From Theorem 2.1, we have the following corollary.

**Corollary 2.1** System (1) with single time delay in state (i.e., \( E_2 = 0, \ldots, E_n = 0 \)) and parameter uncertainties (2)-(3) is quadratically stable with the cost function (4) satisfying the upper bound (6) if the following uncertainty-free system

\[ x(k + 1) = A_0x(k) + A_1x(k - d_1) + \sqrt{\varepsilon}Hw(k), \quad (12) \]
\[ z(k) = \begin{bmatrix} \frac{1}{\sqrt{\varepsilon}}E_0 & 0 \\ 0 & \frac{1}{\sqrt{\varepsilon}}E_1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{1}{\sqrt{\varepsilon}}E_0 & 0 \\ 0 & \frac{1}{\sqrt{\varepsilon}}E_1 \end{bmatrix} x(k - d_1) \quad (13) \]

is asymptotically stable with \( H_\infty \) norm-bound \( \gamma = 1 \).

From Corollary 2.1, it can be seen that the LMI (5) can be viewed as an \( H_\infty \) type condition for an uncertainty-free system.

### 3 State Feedback Synthesis

In this section we consider the problem of robust guaranteed cost control via state feedback for a discrete-time system with multiple time delays and parameter uncertainties. The generalized discrete time-delay system under consideration is described by the state equations:

\[
\begin{cases}
  x(k + 1) = (A_0 + \Delta A_0)x(k) + \sum_{i=1}^{n} (A_i + \Delta A_i)x(k - d_i) + (B + \Delta B (k))u(k), \\
  x(k) = \phi(k), \quad k \in \{-d, -d + 1, \ldots, 0\}, \quad d = \max\{d_1, d_2, \cdots, d_n\},
\end{cases}
\quad (14)
\]
where \( x(k) \in \mathbb{R}^n \) is the state, \( u(k) \in \mathbb{R}^m \) is the control input, \( A_0, A_i (i = 1, 2, \ldots, n) \) and \( B \) are known real constant matrices of appropriate dimensions which describe the nominal system, \( \Delta A_0, \Delta A_i (i = 1, 2, \ldots, n) \) and \( \Delta B \) are real-valued matrix functions representing the time-varying parameter uncertainties. \( d_i (i = 1, 2, \ldots, n) \) is positive integer for time delays and \( \phi(k) \) is an initial state value at time \( k \). The parameter uncertainties considered here are of the form:

\[
\left[ \begin{array}{cccc}
\Delta A_0 & \Delta A_1 & \cdots & \Delta A_n \\
\end{array} \right] \Delta B = HF(k) \left[ \begin{array}{cccc}
E_0 & E_1 & \cdots & E_n & E_{n+1} \\
\end{array} \right],
\tag{15}
\]

where \( H, E_i, (i = 0, 1, \cdots, n + 1) \) are known as real constant matrices of appropriate dimensions which capture the structure of the uncertainties. \( F(k) \) from the set \( \Omega \) similar to that as in (3), that is:

\[
\Omega = \{ F(k) : \sigma_{\text{max}}(F(k)) \leq 1, \quad k \geq 0 \}. \tag{16}
\]

Associated with the system (14) is the following quadratic cost function:

\[
J = \sum_{k=0}^{\infty} [x^T(k)Qx(k) + u^T(k)Su(k)], \quad Q = Q^T > 0, \quad S = S^T > 0. \tag{17}
\]

Without loss of generality, the matrices \( Q \) and \( S \) can be factored as \( Q = D^TD, \quad S = M^TM \).

We consider the following robust guaranteed cost control problem associated with system (14):

Find a controller in the following form:

\[
u(k) = Kx(k) \tag{18}\]

such that the closed-loop system (14) is uniformly asymptotically stable and give an upper bound for the cost function (17) for all admissible uncertainty satisfying (15)-(16).

We have the following theorem for robust guaranteed cost control using memoryless state feedback.

**Theorem 3.1** Consider system (14) with uncertainties satisfying (15)-(16), then there exists a memoryless state feedback controller (18) that solves the addressed robust guaranteed cost control problem if there exist symmetric positive-definite \( X, R_i \) \( (i = 1, 2, \cdots, n) \), a real matrix \( Y \) and a scaling parameter \( \varepsilon > 0 \) such that the following LMI is feasible:

\[
X - \varepsilon HH^T > 0, \quad \text{and} \quad X - \sum_{i=1}^{n} R_i > 0
\]
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\[
\begin{bmatrix}
X - \varepsilon HHT & A_0X + BY & A_{dx} & 0 & 0 & 0 \\
(A_0X + BY)^T & X - \sum_{i=1}^{n} R_i & 0 & XD^T & Y^T M^T & (E_0X + E_{n+1}Y)^T \\
A_{dx}^T & 0 & R & 0 & 0 & E_{dx}^T \\
0 & DX & 0 & I & 0 & 0 \\
0 & MY & 0 & 0 & I & 0 \\
0 & E_0X + E_{n+1}Y & E_{dx} & 0 & 0 & \varepsilon I
\end{bmatrix} > 0,
\]

where

\[A_{dx} = \begin{bmatrix}
A_1X & A_2X & \cdots & A_nX
\end{bmatrix} , \quad E_{dx} = \begin{bmatrix}
E_1X & E_2X & \cdots & E_nX
\end{bmatrix} \]

\[R = \text{diag}(R_1, R_2, \cdots, R_n).\]

Moreover,

(i) the memoryless state feedback control law is given by

\[u(k) = YX^{-1}x(k); \quad (20)\]

(ii) the cost function (17) satisfies the following bound

\[J(k) \leq X^T(0)X^{-1}x(0) + \sum_{i=1}^{n} \sum_{j=-d_i}^{-1} x^T(j)X^{-1}R_iX^{-1}x(j). \quad (21)\]

**Proof.** It can be carried out by using a similar approach as that used in the proof of Theorem 2.1.

>From inequality (6), we can see that the bound of the cost function depends on the initial condition \(\phi(k)\). This dependence can be removed by assuming \(\phi(k)\) is a zero mean random variable such that \(E[\phi(k)\phi^T(k)] = I_n\) \((k = -d, -d + 1, \cdots, 0)\). Then the expectation of the cost function is

\[\bar{J} = E\left[\sum_{k=0}^{\infty} (x^T(k)Qx(k) + u^T(k)Su(k))\right],\]

and the bound becomes

\[\bar{J} \leq E(J) = \text{trace}(X^{-1}) + \sum_{i=1}^{n} d_i \text{trace}(X^{-1}R_iX^{-1}). \quad (22)\]

>From Theorem 3.1, we have the following corollary immediately.

**Corollary 3.1** Consider system (14) with cost function (17), then the optimal value of the cost bound (22) is given by

\[J^* = \inf \left\{\text{trace}(X^{-1}) + \sum_{i=1}^{n} d_i \text{trace}(X^{-1}R_iX^{-1})\right\} \quad (23)\]

\[X > 0, \quad R_i > 0 \text{ satisfying } (19).\]
Moreover, for any given $\mu > \gamma^*$, there exist matrices $X > 0$, $R_i > 0$, $Y$ and a scalar $\varepsilon > 0$ such that the controller defined by inequality (18) is a quadratic guaranteed cost control.

**Remark 3.1** Theorem 3.1 provides an LMI based method for the design of linear memoryless state feedback control laws that robustly stabilize the uncertain discrete delay system of the form (14)-(16) and guarantee an adequate level of performance. It should be pointed out that the feasibility of LMI (19) does not provide a unique solution $X; R_i; Y$ and $\varepsilon$. However, the non-uniqueness is in fact an advantage since we can construct a family of controllers using this property. The optimal quadratic guaranteed cost controller can be determined by searching over all parameters $X; R_i; Y$ and $\varepsilon$ satisfying LMI (19). The optimal cost can be found by Eq.(23) in Corollary 3.1.

**Remark 3.2** The advantage of LMI approach is that the problem of finding the optimal cost can be easily solved without requiring the tuning of any parameters. Indeed, the smallest value of $J$ can be computed by solving the following quasi-convex optimization problem in $X; R_i; Y$ and $\varepsilon$.

\[
\begin{align*}
\text{minimize} & \quad J \\
\text{subject to} & \quad X > 0, R_i > 0, \varepsilon > 0, \text{ and Eq.(19)}. \\
\end{align*}
\]

4 Illustrative Example

In this section we present the same example as used in [5]. Consider system (14)-(16) with the following given parameters and the cost function:

$$
A_0 = \begin{bmatrix}
0.95 & 0.78 \\
0.76 & 0.87
\end{bmatrix}, \quad A_1 = \begin{bmatrix}
0.12 & 0.09 \\
0.11 & 0.07
\end{bmatrix}, \quad B = \begin{bmatrix}
0.5 \\
0.45
\end{bmatrix},
$$

$$
H_1 = \begin{bmatrix}
0.1 & 0.2 \\
0.09 & 0.1
\end{bmatrix}, \quad H_2 = \begin{bmatrix}
0.11 & 0.19 \\
0.1 & 0.09
\end{bmatrix}, \quad E_0 = \begin{bmatrix}
0.2 & 0.17 \\
0.15 & 0.12
\end{bmatrix},
$$

$$
E_1 = \begin{bmatrix}
0.08 & 0.05 \\
0.6 & 0.07
\end{bmatrix}, \quad E_2 = \begin{bmatrix}
0.1 \\
0.09
\end{bmatrix}, \quad \phi(k) = \begin{bmatrix}
e^{-\frac{\mu}{2}} \\
0
\end{bmatrix},
$$

$$
J = \sum_{k=-d_1}^{1} [x^T(k)Qx(k) + u^T(k)Su(k)], \quad Q = \begin{bmatrix}
0.1 & 0.05 \\
0.05 & 0.09
\end{bmatrix}, \quad d_1 = 6, \quad S = 0.05.
$$

Also, we factor $Q$ and $S$ such that $Q = D^TD$, $S = M^TM$, where

$$
D = \begin{bmatrix}
0.3048 & 0.0844 \\
0.0844 & 0.2879
\end{bmatrix}, \quad M = 0.2236.
$$

By some manipulation, we can get that

$$
\widetilde{H}_1 = H_1 = \begin{bmatrix}
0.1 & 0.2 \\
0.09 & 0.1
\end{bmatrix}, \quad \widetilde{E}_1 = \begin{bmatrix}
0.015 & 0.0475 \\
0.6065 & 0.0702
\end{bmatrix}.
$$
Solving the LMI (19), one obtains

\[
X = \begin{bmatrix} 0.8712 & -0.1744 \\ -0.1744 & 1.1607 \end{bmatrix}, \quad R = \begin{bmatrix} 0.5153 & -0.1298 \\ -0.1298 & 0.5430 \end{bmatrix} \\
Y = \begin{bmatrix} -1.2023 \\ -1.5427 \end{bmatrix}, \quad \varepsilon = 1.183,
\]

and therefore by (20) the control parameter \( K \) can be calculated as

\[
K = \begin{bmatrix} -1.6971 \\ -1.5840 \end{bmatrix}.
\]

With the above control parameters, the state feedback control law (20) robustly stabilizes uncertain system (14)-(16) and guarantees an adequate level of performance

\[
J(k) \leq 16.6058.
\]

However, the value of the cost bound in [5] is \( J(k) \leq 22.0328 \), which shows the improvement of the approach proposed here comparing with that used in [5].

5 Conclusions

In this paper, the problem of robust guaranteed cost control for discrete multiple time-delay systems with time-varying uncertainties has been solved by memoryless state feedback. A sufficient condition for guaranteeing not only the quadratic stability of the closed-loop system but also the cost function bound constraint is presented. Moreover, a simple form cost bound is given by assuming the initial state values to be a zero mean random variable. An numerical example is included to demonstrate the theoretic results obtained.

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References


