CONSEQUENCE AS INFERENCE

Mediæval Proof Theory 1300–1350

The first half of the fourteenth century saw a remarkable flowering in accounts of consequences (*consequentiae*). Logicians began to write independent treatises on consequences, the most well-known being those by Walter Burleigh (*De consequentiis*) and Jean Buridan (*Tractatus de consequentiis*). Consequences also came to be treated systematically in comprehensive works on logic, such as those of Walter Burleigh (both versions of the *De puritate artis logicae*), William of Ockham (*Summa logicae*), and, to a lesser extent, Jean Buridan (*Summulae de dialectica*)—as well as in works written in their wake.[1] The philosophical achievement realized in these various writings was no less than a formulation of a theory of inference: the rules for consequences given by these mediæval authors spell out a natural deduction system in the sense of Jaskowski and Gentzen.[2]

[1] All translations are mine. I what follows I cite the Latin text only when it is not readily available (*e. g.* for much of Buridan's *Summulae de dialectica*), when there is a textual difficulty, or when a point depends on its original phrasing. The texts on which this study is based are all listed in Part (A) of the Bibliography; when I speak of "the available literature" these are the works I have in mind. Green-Pedersen [1983] catalogues several other texts about consequences that exist only in manuscript. The available literature seems to fall roughly into four groups. [1] The anonymous two earliest treatises on consequences, along with Walter Burleigh's *De consequentiis* and his *De puritate artis logicae*—the longer version being influenced by Ockham. [2] William of Ockham's *Summa logicae*, whose influence can be seen in the *Elementarium logicae* and the *Tractatus minor logicae* (formerly ascribed to Ockham himself), the anonymous treatises *Liber consequentiarum* and *Logica "Ad Rudium"*, and the unusual commentary *De consequentiis* possibly written by Bradwardine. [3] Jean Buridan's *Tractatus de consequentiis* and *Summulae de dialectica*, whose influence can be seen in Albert of Saxony's *Perutilis logica* or Marsilius of Inghen's *De consequentiis*. [4] The *Consequentiae* of Robert Fland and of Richard Ferrybridge, dating from the close of the first half of the fourteenth century, which have many affinities with the later works of (for example) Richard Billingham, Richard Lavenham, and Ralph Strode.

[2] See Jaskowski [1934] and Gentzen [1935]; Prawitz [1965] gives a modern presentation of natural deduction systems. The claim defended here is the mediæval counterpart of the case put forward for Aristotle's logic initially by Smiley [1973] and Corcoran [1974], since developed in Lear [1980], Thom [1981], and Smith [1989]. Note the limited scope of my thesis: whatever consequences may have been before 1300 and whatever they may have become after 1350, in the first half of the fourteenth century they constituted a natural deduction system. An admirably clear statement of this position is given in Moody [1953] 15: "The theory of consequence, taken as a whole, constituted a formal specification of inference-conditions for the formulated language."

Recognition that mediæval logicians are dealing with inference in the theory of consequences, rather than with implication, is sporadic at best and nonexistent at worst.[3] This may be due to the emphasis many modern logicians put on presenting logical systems axiomatically, since axiomatic formulations typically have only a single rule of inference (detachment) and focus on logical truth instead of logical consequence.[4] But whatever the cause, the point that consequences are inferences has not been appreciated, which in turn has made it hard to see how consequences fit into the mediæval conception of argument.

The discussion will proceed as follows: §1 argues that consequences are not the same as conditionals; §2 considers two objections to this distinction; §3 argues that consequences are inferences and were understood by mediæval logicians to be so; §4 examines accounts of formal validity; §5 looks at the place of consequences—the theory of inference—in their general account of argumentation. I'll draw some morals about the mediæval logical enterprise by way of conclusion.

1. Implication and Consequence

A categorical sentence (say) is used to make a statement, that is, "to say something of something" in Aristotle's phrase.[5] Conditional sentences also make statements, that is, they declare that a certain relation obtains (namely that the consequent is conditional upon the antecedent). The statement that a conditional sentence makes is not the same as the statement

Kneale & Kneale [1962] 4.5 describe "a change of fashion" in writings on consequences around 1300, "something like that from Aristotle's presentation of syllogistic theory by means of conditional statements to Boethius' presentation by means of inference schemata" (277). In these historians natural deduction has been glimpsed, but only as in a mirror darkly; I intend to show it to the reader face-to-face.

[3] Three recent examples, each a near miss. Boh [1982] 300 writes: "Implication, entailment, and inference are all distinct from one another... Nevertheless, medieval logicians disconcertingly use the single notion of consequence to cover all three of these relationships between propositions." (They did no such thing, as we shall see in §§1–3 below.) In King [1985] 59–60 I argued that consequences have features of conditionals as well as inferences, and hence are neither fish nor fowl. Adams [1987] 458–490, who quite properly renders *consequentiae* as 'inferences', discusses at length whether Ockham's rules define a version of strict implication.

[4] Most modern interpretations of mediæval rules for consequences take them at best to present axioms, or perhaps theorems, of a connexive logic (as in MacCall [1966]).

[5] Aristotle, *De interpretatione* 5 17^a21-22 (Boethius's translation): "Harum autem haec quidem simplex est enuntiatio, ut aliquid de aliquo uel aliquid ab aliquo..." See also *De interpretatione* 6 17^a25-26: "Affirmatio uero est enuntiatio alicuius de aliquo, negatio uero enuntiatio alicuius ab aliquo."

made by any of its parts taken in isolation, of course; conditionals neither say what their antecedents or their consequents say, nor are they about the subjects of their antecedents or consequents. For all that, conditional sentences do succeed in making statements. Inferences, however, do not "say something of something." They do not make statements. An inference is a performance: it is something we do, perhaps with linguistic items, but in itself it is no more linguistic than juggling is one of the balls the juggler juggles. Furthermore, even the statement of an inference (its linguistic representation) is not a statement-making expression. It has parts that in isolation could be used to make statements—namely any of the premisses or the conclusion—but itself does not make a statement. (One sign of this is that neither an inference nor the statement of an inference is assessed as true or false.) In a slogan: conditionals make statements whereas inferences do things with statements.[6]

Modern logicians regiment this distinction between conditionals and inferences by presenting them as categorically different parts of the logical landscape: the former through a primitive or defined sentential connective appearing in well-formed formulae, for which truth is appropriate, and the latter through rules for transforming well-formed formulae, for which validity is appropriate. Thus conditionals and inferences differ in kind, one belonging to the object-language and the other to the metalanguage. They are not unrelated, however; a Deduction Theorem can be established for many axiomatic systems, so that if $A \rightarrow B$ then $A \vdash B$,[7] and natural deduction systems typically use conservative introduction and elimination rules to define the conditional connective.

Mediæval logicians, like their modern counterparts, treat implication

[6] This account oversimplifies the complex nature of conditionals, even the "ordinary" present-tense indicative conditional. See Woods [1997] or the articles in Jackson [1991] for an account of some of the difficulties. There are other reasons for distinguishing conditionals from inferences; the argument in Carroll [1895] shows that axioms need to be supplemented by rules of inference. Haack [1976] argues that the need for a justification of deduction outlined in Dummett [1973] generalizes Carroll's argument into a dilemma, so that there is either an infinite regress or circularity. But these arguments were unknown in the fourteenth century, so I will not treat them here. (This is not to say that mediæval logicians did not recognize the need to justify particular inference-rules; they surely did, but just as surely didn't see the enterprise of doing so as deeply problematic.)

[7] The Deduction Theorem can be proved from the following two axioms: $A \rightarrow (B \rightarrow A)$ and $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$ along with detachment, by induction on proof length. Modern logical systems sharply distinguish syntactic consequence ($\vdash$) from semantic consequence ($\models$); the mediæval analogue is discussed in §4.

and inference as logically distinct notions, along the lines sketched above. To begin at the beginning: a conditional sentence is a particular kind of statement-making utterance, but different in kind from the paradigm case of the (simple) categorical sentence.[8] It is instead lumped together with conjunctive and disjunctive expressions under the generic heading of "compound sentence" (*propositio hypothetica*), on the grounds that these three kinds of utterance all have parts that would qualify as sentences taken by themselves although they are not simply reducible to their parts—a mediæval version of our notion of the connective of widest scope.[9] So much is commonplace, derived from Boethius and ultimately from Aristotle (*De interpretatione* 5 17^a9–10 and 20–22). Another mediæval commonplace is that logic is divided into three parts, namely into words, statements, and arguments, ordered by composition: statements are made out of words, and arguments out of statements. These parts are not reducible to one another, for we use words to make statements and we use statements to make arguments.[10] Each part of logic thus constitutes its own level of analysis and carves out a distinct part of the logical landscape. Conditional sentences, as statement-making utterances, must therefore differ in kind from arguments, since they belong to different parts of the landscape. Hence consequences will be distinct from conditionals—at least, to the extent that mediæval logicians classify consequences with arguments.

The strength of this line of reasoning lies precisely in its premisses being commonplace. It does not depend on any particular feature of the doctrine of consequences. We can reason our way to the categorical distinction between implication and inference from entrenched mediæval views about

[8] A sentence may be categorical whether it be affirmative or negative (the quality of a sentence is part of its logical form); universal, particular, or indefinite (so too the quantity); assertoric or modal; even—within limits—internally complex.

[9] Negation is not a connective: sentential negation is accomplished by a categorically distinct copula, so that 'is' and 'is not' are two different functors (*mutatis mutandis* through all the tenses and modes). The mediæval account of compound sentences doesn't precisely match the modern notion of widest scope, since the latter has mathematical properties the former lacks, *e. g.* embedding of formulae at arbitrary depth. The precise kind of statement made by compound sentences is a matter of some complexity. We need not explore it here. It is not the details but the bare fact that conditional sentences make statements (and thereby are true or false) that is significant.

[10] More exactly: sentences consist in words but are not simply sequences of words; the combination of words into a sentence used to make a statement goes beyond anything in the words themselves—sentences are a way of doing something with words (namely making a statement). Likewise, arguments are a way of doing something with statements, as noted above.

logic and language. The only question that remains is whether consequences and arguments do belong together.

In comprehensive works on logic, where systematic concerns readily come to the fore, consequences are classified with arguments and not with sentences. William of Ockham provides a clear example. The *Summa logicae* is organized into three parts based on the division of logic recounted above. Conditional sentences are treated in *Summa logicae* 2.31 (devoted to sentences) as a species of compound sentence, whereas consequences are the subject of the third treatise of *Summa logicae* 3 (devoted to arguments).[11] Ockham even refers to the later discussion of consequences in his brief chapter on conditionals, so he is aware of the distinction at precisely the point at which it matters.[12] The fragmentary nature of Walter Burleigh's *De puritate artis logicae* in both versions makes it less useful as evidence, but he does describe consequences as rules (60.12-14) and not as sentences. Jean Buridan doesn't have a separate discussion of consequences in his *Summulae de dialectica*,[13] but at the beginning of his *Tractatus de consequentiis* 3 he classifies all arguments as species of consequence.

These architectonic considerations give us some presumptive evidence that consequences are logically grouped with arguments rather than sentences. They cannot do more than that, since such considerations do not rule out the possibility that conditionals are a species of consequence—that the term *consequentia* was used to describe arguments and to describe conditionals. But there is both negative and positive textual evidence against this objection, in support of the claim that mediæval logicians not only recognized a difference between implication and inference but found them not to overlap at all.

[11] The third part analyzes arguments in general, as Ockham tells us in *Summa logicae* 3-1.1 (359.2-3).

[12] The anonymous *Logica ad Rudium*, structured in the same fashion as the *Summa logicae*, likewise treats conditionals as a kind of compound sentence (2.76-78) and consequences as a form of argument (3.64-84). So too the *Tractatus minor logicae* 2.2 for conditionals and Book 5 for consequences, as well as the *Elementarium logicae* 2.16 and Book 6. Now Ockham and other logicians sometimes restrict consequences to nonsyllogistic inferences, but this is a matter of terminology and not doctrine: Ockham expressly says that it is a terminological convenience.

[13] Buridan's *Summulae de dialectica* is divided into the following treatises: [1] introductory material and sentences; [2] predicables; [3] categories or categorematic expressions; [4] supposition; [5] syllogisms; [6] dialectical topics; [7] fallacies; [8] demonstrations; and sometimes [9] sophisms. Although there is no treatise devoted to consequences, Buridan does discuss them in [5]–[6], whereas he describes and defines conditionals in [1].

The negative evidence is as follows. In all the available literature of this period, which runs to hundreds of pages, I have found no instance of any author treating 'conditional' and 'consequence' as synonymous. Nowhere does the expression *consequentia seu condicionalis* or the like occur.[14] Of course, the architectonic considerations given above suggest that these terms would not be everywhere interchangeable. But they might well be interchangeable in certain contexts. For example, when speaking of conditionals proper, some feature that they have in common with consequences generally might be under investigation. It is striking, though not conclusive, that such expressions are never employed even in such contexts.

The positive evidence comes in two varieties. First, the authors under consideration not only resist treating the terminology as interchangeable, they also use it to mark a logical distinction: conditionals do not merely appear along with consequences; they are actively contrasted with consequences. Second, conditionals have different properties, since they are true or false whereas consequences are not.[15] We'll take each in turn.

The evidence for the first claim is as follows. Walter Burleigh mentions conditionals and consequences together in his *De consequentiis* §8, where he is talking about the legitimacy of inferring a conditional composed of the consequent of the last of a string of conditionals from the antecedent of the first of the string, that is: $A \to B, B \to C \vdash A \to C$. This classic example of cut-elimination, which Burleigh calls "the start-to-finish inference" (*consequentia a primo ad ultimum*), also appears in both versions of his *De puritate artis logicae* (70.1–23 and 200.20–201.3). In each instance Burleigh explicitly contrasts the conditional sentences that enter into such reasoning with the consequence made out of them. Furthermore, Burleigh devotes *De consequentiis* §§66–72 to consequences that hold among conditionals, clearly assigning different properties to each. For example, he gives truth-conditions for conditional sentences (§68), contrasting them with consequences, which hold in virtue of topics (§§71–72).[16] The parallel section

[14] The only possible exception: in *Summulae de dialectica* 7.4.5 (discussed in §5), Buridan does say that 'consequence' can mean either a conditional sentence or an inference. But his entire discussion of consequences uses the second sense, not the first, which he never mentions again.

[15] Consequences in fact have a distinct property: they can be legitimate, and thereby they may hold or be valid. This will be discussed in §§3-4.

[16] In §69, Green-Pedersen renders the text "Exemplum primae: si tu es Romae, ergo illud quod est falsum est uerum..." adding the '*ergo*' with O (rather than omitting it with CL). But Burleigh is giving here an example of a conditional, not a consequence; the '*ergo*' should be suppressed.

in the longer version of the *De puritate artis logicae* (66.9-79.10) reiterates these claims.

Other logicians also contrast conditionals with consequences. William of Ockham, as noted above, refers us in his chapter on conditional sentences (*Summa logicae* 2.31) to his later discussion of consequences. The commentary *De consequentiis* on Ockham, possibly written by Bradwardine, declares in §7 that we can move from a consequence $A \vdash B$ to a conditional $A \rightarrow B$; Rule 15 of the *Liber consequentiarum* says that we can move in either direction (a claim to be explored more fully in §5 below). Richard Lavenham in his late work *Consequentiae* §§41–47 gives seven rules describing consequences that hold among conditionals. The claims put forward by these philosophers would not make sense unless consequences were something other than conditionals.

The evidence for the second claim is as follows. Burleigh and Ockham, for example, explicitly call conditional sentences true or false: see the *De consequentiis* §68 and *Summa logicae* 2.31 respectively. Likewise for the *Logica ad rudium* 2.76, the *Tractatus minor logicae* 2.2, and the *Elementarium logicae* 2.16. Even Buridan calls conditionals true in his *Summulae de dialectica* 1.7.3.[17] Of course, the fact that conditionals may be true or false follows from the fact that conditional sentences are statement-making utterances, since what it is to be a statement is, at least in part, to have a truth-value (*De interpretatione* 4 17^a3–4)—putting aside for now worries about future contingents and other puzzling cases.

Consequences, on the other hand, are neither true nor false. Here the negative evidence has quite a bit of weight. In the hundreds of pages of the available literature there are countless opportunities to say of consequences that they are true or false, opportunities that are all the more pressing since the writers are usually grappling with the question which consequences are to be approved and which not. Yet in all these pages I know only three passages in which consequences are called true or false.[18] One occurs in Pseudo-Scotus and is a mere slip.[19] But the other two are found in the

[17] "Notandum est quod haec est una condicionalis uera et necessaria: si homo est asinus, homo est animal brutum."

[18] A fourth passage can be set aside as merely terminological. In the second *Anonymi de consequentiis* §19 (Green-Pedersen [1980]), mention is made of a 'false consequence'—an instance of asserting the consequent—but this is plainly an extension of 'false' to inferences that are fallacious, not meant to ascribe a truth-value; it is no more to be taken seriously than Burleigh's willingness to speak of the same fallacy as being a "false rule" in both versions of his *De puritate artis logicae* (200.16–17 [shorter] and 62.14–15 [longer]).

[19] In speaking of material consequences, Pseudo-Scotus says that some are true *sim-*

writings of Jean Buridan, a logician of the first rank. Now Buridan's view of consequences might simply be idiosyncratic; we could set his testimony aside, given that there is no similar evidence in any other author of the period. But we do not have to do so. The context of each passage shows that we are not to take seriously Buridan's mention of truth-values in connection with consequences.

The first passage is found in Buridan's *Tractatus de consequentiis* 1.3.4–6 (21.16–25), where he points out that some people (*aliqui*) say that any hypothetical sentence formulated with 'if' or 'therefore' is a consequence and that there are thus two kinds, namely true and false consequences; he replies:

> In this treatise—whether or not it be true—words signify by convention; I mean to understand by 'consequence' a true consequence, and by 'antecedent' and 'consequent' sentences one of which follows from the other by a true or legitimate consequence (*uera seu bona consequentia*).

Buridan concedes the terminology to the unnamed thinkers whose view is under discussion. He stipulates what he will call a consequence after reminding us of the conventionality of language, characterizing consequences not only as true but as "true or legitimate"—and then never calling them 'true' again in the rest of his treatise (a treatise devoted to consequences, mind you!).[20] This passage therefore cannot serve as evidence that consequences, like conditionals, have truth-value.

The second passage is found in Buridan's *Summulae de dialectica* 1.7.6:[21]

> It seems to me that a hypothetical sentence joining together two categorical sentences by 'therefore' should likewise be counted as

pliciter and some *ut nunc*—the former can be reduced to formal consequences by the assumption of a necessary proposition, whereas the latter refer to consequences that hold contingently, not at all times. Yet by the time he gets to *ut nunc* consequences he switches back to speaking of legitimacy, never returning to truth again. (See the translation included in this volume.) It seems clear that this is no more than a slip of the pen, since nothing in his discussion turns on whether legitimacy or truth is at stake and the usage is completely isolated.

[20] Elizabeth Karger has proposed that Buridan is here stipulating that he will use 'consequence' to pick out only true conditionals, and hence that consequences do have truth-value. This reading is possible, but, I think, not borne out by other evidence: nowhere else in the *Tractatus de consequentiis* does Buridan ever rely on consequences having truth-value. The passage is surely anomalous.

[21] "Et uidetur mihi quod talis hypothetica coniungens categoricas per 'ergo' debet similiter reputari falsa si non sit necessaria consequentia, quae designatur per istam dictionem 'ergo', et quod etiam sit falsa simpliciter loquendo si habeat aliquam praemissam falsam."

> false if the consequence is not necessary (which is denoted by the word 'therefore'!), and also that it is false simply speaking if it were to have some false premiss.

Buridan proposes that we count an argument as false (*reputari*) if it fails to establish its conclusion by being either invalid or unsound. Yet there is no suggestion here that a consequence is literally true or false the way a statement must be. On the contrary, Buridan's plain meaning is that consequences can be invalid or unsound, and that these are defects in consequences just as falsity is a defect in a statement.

To sum up: in neither passage does Buridan seriously propose that conseqences have truth-value; even if he were to do so, we can oppose to this the negative testimony of the rest of his writings, wherein consequences are not called true or false. And, as remarked above, even if Buridan were to allow consequences to have truth-value, no other logician in this period does. Instead, they say that consequences are "legitimate (*bona*)" and that they "hold (*tenet*)" or "are valid (*ualet*)"—properties explored further in §4.[22]

So much for the positive evidence that mediæval logicians recognized the distinction between consequences and conditionals. The story is not complete, of course; to say that consequences aren't conditionals does not establish what they in fact are. Before presenting the positive case that consequences are inferences, though, we need to look at two objections to the thesis that conditionals and consequences are logically distinct notions.

2. Two Objections

The first objection runs as follows. Conditional sentences are made up of parts, namely the antecedent and the consequent. Similarly, arguments are made up of parts, statements that we call the premiss(es) and the conclusion. But the parts of consequences are uniformly called the 'antecedent' and the 'consequent' throughout the available literature. Hence consequences must be a form of conditional sentence rather than of argument.

The factual claims in this objection are correct, but the conclusion that consequences must be a form of conditional sentence does not follow. The mistake here is easy to make. The Latin terminology is *antecedens* and

[22] The *Elementarium logicae* 2.16 contrasts consequences and conditionals by their possession of different properties (94.3–8): "Just as a consequence can be legitimate even though neither of its sentences is true, and even though the antecedent is false and the consequent true—but is never legitimate if its antecedent is true and its consequent false—so too a compound conditional sentence can be true even if neither of the categorical sentences of which it is composed is true, and even if the first is false and the second true, but not if the first is true and the second false."

consequens, the ancestors and cognates of the English words 'antecedent' and 'consequent.' Modern logicians regiment their use so that they properly apply only to conditional sentences.[23] Well, they do apply to conditionals in Latin, but they are not tied to them the way the Greek grammatical terms *protasis* and *apodosis* are—that is, unlike the Greek terms, the Latin terms are not simply defined relative to one another by their occurrence in conditional sentences. Instead, *antecedens* and *consequens* carry the broader senses of 'what comes before' and 'what comes after.'[24] Hence they are equally applicable to the parts of conditional sentences and to consequences. The *De consequentiis* possibly written by Bradwardine says so explicitly (§2):[25]

> Note that a consequence is an argumentation made up of an antecedent and a consequent.

Consequences are arguments, and, as arguments, they have two logically distinct parts: one that comes before (the antecedent) and one that comes after (the consequent). The terminology is more general than that of premisses and conclusion, but no less legitimate. The pull of the cognate word and its restricted English sense is hard to resist. But resist it we should. The terminology used to talk about consequences doesn't give us any reason to interpret them as conditionals, although it tempts us to.

The second objection is this. Buridan explicitly says that consequences are compound sentences—*Tractatus de consequentiis* 1.3.2 (21.9–10):[26]

> There are two types of sentence, namely categorical and compound, and a consequence is a compound sentence made up of many sentences joined together by 'if' or 'therefore' or their equivalent.

Yet we have seen above that there are only three kinds of compound sentence, namely disjunctive, conjunctive, and conditional sentences. Consequences are surely neither of the first two, and hence must be identified as a

[23] Modern logicians have been largely successful with 'consequent' (although it sometimes carries the sense of 'important' outside logical circles) but not at all with 'antecedent', which still has a broad range of uses not tied to either conditionals or consequents—for instance, in speaking about one's background or genealogy.

[24] See *The Oxford Latin Dictionary* [1982] at 138AB *s. v.* antecedo, 413BC *s. v.* consequens/consequor. Note that the constituents of a Gentzen sequent in a natural deduction system are called the 'antecedent' and the 'succedent'—acceptable translations of *antecedens* and *consequens*!

[25] Burleigh refers explicitly to the "syllogistic antecedent" (*antecedens syllogisticum*) of a consequence in the longer version of his *De puritate artis logicae* (65.7).

[26] Buridan repeats the point at *Tractatus de consequentiis* 1.3.12 (22.61). The same claim is made in passing by the Pseudo-Scotus at the start of q.10; see also Pinborg [1972] 170.

kind of conditional sentence. Thus consequences belong to the same part of the logical landscape as conditionals, and so we can reject the presumptive evidence explored in §1 in favor of distinguishing them.

There are two replies available to this second objection.[27] First, while it is true that Buridan calls consequences compound sentences, he also calls the syllogism—the paradigm case of an argument—a compound sentence, and in fact reducible to a conditional sentence.[28] If syllogisms are reducible to conditional compound sentences, consequences can still be identified with arguments, although we may have to redraw the line between (compound) sentences and arguments in some fashion. Given that Buridan classifies all arguments as species of consequence, as noted in §1, it would follow that the distinction between arguments and non-arguments would have to be made among kinds of conditional sentences.

The drawback to this first reply is that it would require us to admit that consequences (and arguments generally) are in fact conditionals, which was the problem the reply was supposed to avoid. However, it does suggest that the way to approach the second problem is by considering what might have led Buridan to think that consequences were sentences in the first place. Recall from the start of §1 the slogan that conditionals make statements whereas inferences do things with statements. True enough, but we can also describe the inferences that we make, and we do so with sentences describing how we manipulate statements. Here is one: "All swans are white objects; therefore, some white objects are swans." What kind of sentence is this?

Modern logicians would say that this sentence does not belong to the object-language, despite its similarity in surface grammar to, say, the conditional sentence "If all swans are white objects, then some white objects are swans." The inference has the logical form $A \vdash B$ (rather than $A \rightarrow B$); the turnstile '$\vdash$' acts as a kind of metalinguistic connective. Hence $A \vdash B$

[27] A third reply—that Buridan can be discounted as a single voice against many others—will not do for two reasons. First, Buridan was a superb logician, and voices must be weighed rather than counted. Second, he may not be a lone voice; the longer version of Burleigh's *De puritate artis logicae* seems to classify consequences under the generic heading of conditional compound sentences (the first part of the second treatise), although the incomplete nature of the text makes it hard to put much weight on its structural divisions.

[28] *Summulae de dialectica* 5.1.3: "Respondeo quod licet syllogismus sit compositus ex pluribus orationibus, tamen est una propositio hypothetica, coniungens conclusionem praemissis per hanc coniunctionem 'ergo'. Et potest reduci ad speciem propositionum condicionalium, quia sicut condicionalis est una consequentia, ita et syllogismus; unde syllogismus posset formari per modum unius condicionalis sic 'si omne animal est substantia et omnis homo est animal, omnis homo est substantia'."

and $A \rightarrow B$ are not on a par.

Mediaeval logicians do not have our distinction between object-language and metalanguage. A sentence representing an inference is on all fours with a conditional sentence, or any other sentence for that matter. Yet Buridan captures the spirit of the modern reply. He admits that the statement of an inference is a compound sentence. But this admissison is compatible with the claim that the statement of an inference (its linguistic representation) is not a statement-making expression, whereas a conditional sentence is a statement-making expression.[29] Significantly, Buridan only says that consequences are *reducible* to conditional sentences, not that they *are* conditional sentences; Buridan, unlike Ockham and his followers, does not think we can pass from one to the other in any direct fashion (as we shall see in §5). Hence Buridan can reject the trichotomy of choices among compound sentences, on the reasonable grounds that it taxonomizes the kinds of statements that different sentences can make, whereas the sentence describing a consequence does not make a statement at all, but instead describes something done with statements.

3. Consequence and Inference

In the course of disentangling consequences from conditionals we have run across evidence that consequences are arguments, or at least closely related to arguments. From the position they occupy in comprehensive logical treatises to Buridan's classification of argument as a species of consequence to the bald statement in *De consequentiis* §2 cited above, mediæval logicians take pains to underline the inferential force of consequences. Ralph Strode, perhaps in the 1360s, explicitly says that "a consequence is a deduction (*illatio*) of the consequent from the antecedent" (1.1.02).[30] Buridan contrasts conditionals, which are not arguments at all, with consequences in the proper sense, which are indeed arguments (*Summulae de dialectica* 7.4.5). There is also a wealth of secondary evidence that consequences are infer-

[29] There are special challenges for Buridan, as a nominalist, to distinguish sentences (particular mental tokenings perhaps correlated with individual utterances or inscriptions) from the statements they make. We need not pursue this point here.

[30] Consequences are identified either (quasi)-syntactically as sentences connected by an illative particle, or as the relation obtaining among such sentences—for example, in §1 of the first anonymous treatise on consequences in Green-Pedersen [1980], a consequence is defined as a "relationship (*habitudo*) between an antecedent and a consequent". (Note that Green-Pedersen [1983] calls the second definition 'semantic' and says that one or the other is given in all the writings on consequences, that is, in both the published and unpublished manuscript texts.)

ences in the terminology and the proof-procedures employed by logicians and philosophers alike during this period. Let's have a look.

Consequences underwrite arguments.[31] They argue for (*arguitur*) or permit us to draw (*concluditur*) a conclusion from the premiss or premisses, and to say in general what follows (*sequitur*) from what. They can be established (*probatur*) by supporting grounds. Furthermore, they are said either to be valid (*ualet*) or hold (*tenet*), or, if not, to be fallacious (*fallit*). If a consequence is appropriately truth-preserving—a feature to be investigated in §4—then it is said to be legitimate (*bona*).[32] Burleigh is especially clear about these features of consequences in his *De consequentiis*: in §11 he says that when a consequence is legitimate a given conclusion ought to be inferred (*debet inferri*) through it; in §12 he says that a test to see whether a consequence is valid determines whether it is legitimate; in §13 he refers to legitimacy (*bonitas*) as a property suited to consequences, as truth is to sentences, whose presence depends on the inference drawn (*quod inferatur*) in a given case.

Mediaeval philosophers, not just mediæval logicians, recognized in practice that the consequence provides the inferential force of an argument. Typically, after stating an argument, a proof will be offered of each of the premisses, followed by a proof of the consequence (*probatio consequentiae*) to ensure that the conclusion does in fact follow from the premisses. Often the consequence is established by showing that it conforms to an accepted rule, or that its violation would conflict with such a rule. Given true premisses and a valid inference, of course, the result is a sound argument; nothing but the consequence can play the role of the latter.

The rules for consequences found in the treatises of this period spell out the admissible sequents of a natural deduction system. Consider, for example, the first rule for consequences offered by Ockham in his *Summa logicae* 3-3.2 (591.9–11):

> There is a legitimate consequence from the superior distributed term to the inferior distributed term. For example, "Every animal is running; hence every man is running."

Such rules are typically given in metalogical or schematic terms (often in both ways), and they clearly refer to inferences that hold in virtue of the

[31] See §2 of the *De consequentiis* possibly written by Bradwardine: "Every consequence is taken to underwrite* some argument" (**probandum* L; Green-Pedersen adopts *producendum* from the badly defective V).

[32] Mediaeval logicians, like modern logicians, vacillated about whether to say that a fallacious inference was an inference, and hence whether 'legitimate inference' was pleonastic. The sense is usually clear from context. I'll follow the mediæval lead here.

logical form of their constituents. One of the earliest independent works on consequences assimilates the legitimacy of a consequence to its formal validity:[33]

> This rule may be employed for seeing which consequences are legitimate and which not. We should see whether the opposite of the consequent can obtain with the antecedent. If not, the consequence is legitimate. If the opposite of the consequent can obtain with the antecedent, the consequent is not formally valid (*non ualet de forma*).

Hence the rules for consequences determine what inferential moves can be made; at least some rules require that the inferences hold in virtue of the logical form of the statements involved.

Ordinary principles of natural deduction are easily found in the mediæval literature, as one might expect from the 'naturalness' of natural deduction. Walter Burleigh gives a concise formulation of detachment in the longer version of his *De puritate artis logicae* that can be virtually transcribed from the Latin: $A \to B, A \vdash B$ (66.13: *Si A est, B est; sed A est; ergo B est*).[34] Now most of the treatises in this period give cut-elimination among their very first rules: $A \to B, B \to C \vdash A \to C$ (cited for Burleigh in §1 above). Examples could easily be multiplied. Some mediæval rules for consequences have no modern parallel, since they depend on the details of mediæval term-logic and syntactic analysis; Ockham's first rule, cited above, is a handy instance.[35] Likewise, some modern rules of natural deduction have no mediæval parallel, such as those depending on mathematical features of the formulae (recursiveness, arbitrary depth, normal form). Then again, at certain points mediæval logic and modern logic arguably diverge, as perhaps they do over existential import. For the most part, however, there is a remarkable degree of consensus between mediæval rules for consequences and modern natural deduction principles of first-order logic. There is even some agreement between mediæval and modern logic on higher-order deductive principles, namely on the proof-procedure

[33] The first *Anonymi de consequentiis* §18 (Green-Pedersen [1980] 7.12–15).

[34] Detachment is a rule of inference ; thus it is not to be confused with the (related but distinct) law of propositional logic $(A\&(A \to B)) \to B$.

[35] Note that Ockham's first rule treats the relation between the terms 'animal' and 'man' as a formal feature. Modern first-order logic does not normally respect such relationships, but could do so in a number of ways: indexing or sorting the term-variables; adding semantic rules along the lines of meaning-postulates; and the like. Are such consequences formal? How would we decide? What difference does it make? See §4 below.

for establishing the validity of syllogisms other than the first four moods of the first figure: *Barbara*, *Celarent*, *Darii*, *Ferio*. Following Aristotle's lead, mediæval logicians adopted a general *reductio*-method to validate at least some syllogisms (traditionally only *Baroco* and *Bocardo*); Buridan offers a clear statement of it in the third theorem of his *Tractatus de consequentiis* 3.4 (87.99–103).[36] In modern systems Buridan's theorem can be restated as a metalogical rule for classical *reductio*. Naturally, there is no mediæval parallel to other techniques of modern proof theory, many of which are artifacts of the mathematical nature of modern logic (such as induction on proof-length). But when idiosyncratic features of mediæval logic or of modern logic do not intrude, the deduction-rules provided by each system are largely the same.

The rules for consequences, then, spell out a natural deduction system. The elements of this system are inferences—that is, consequences—which can be used to license arguments.[37] Hence the rules for consequences state legitimate inference-schemata. But what makes any inference-scheme legitimate, or even preferable to another? What, if anything, makes inferences valid?

4. Formal Validity

The mediæval consensus on legitimate inference-schemata does not extend to the explanation of legitimacy itself. In the first half of the fourteenth century we find three competing accounts of what makes a consequence legitimate. The first is little more than a suggestion, similar to a modern informal characterization of deductive validity. The second explains the legitimacy of $A \vdash B$ modally, such that it is impossible for A to be true and B false; this account, then as now, is the favored view. But it is not without its problems. Hence a third account, based on substitutivity, was specifically designed to capture formal validity. We'll consider each in turn.[38]

First, Robert Fland opens his *Consequentiae* by giving rules for knowing

[36] I badly mangled the analysis of Buridan's account of the *reductio*-method in King [1985] 73-74 (not least by using conditional form in my account).

[37] There is some looseness here: do we identify the consequence as the inferential force of the argument, or as the argument constituted by the inferential force? (Is the inference the whole formula $A \vdash B$? just $\vdash$? the open formula $\ldots\vdash\ldots$?) Different mediæval authors answered the question differently.

[38] The three accounts canvassed in this section have usually been identified as truth-conditions for implication. As such, they seem to spell out intensional (psychological), modal (strict entailment), and formal conceptions of implication. However, they are accounts of validity rather than truth-conditions, as we shall see.

when consequences are formal, which is the case "when the consequent is understood in the antecedent formally" (§1). This psychological or epistemic account seems resistant to logical treatment, and, on the face of it, more appropriate to characterizing implication—relevant implication at that—than inference. (To say nothing of its circularity!) However, around 1370 Richard Lavenham took up the same train of thought in his *Consequentiae*, and his remarks, though equally brief, give us a clue how to interpret Fland (§2):

> A consequence is formal when the consequent necessarily belongs to the understanding of the antecedent (*necessario est de intellectu antecedentis*), as it is in the case of syllogistic consequence, and in many enthymematic consequences.

The tip-off that we are dealing with inference is seen in Lavenham's mention of syllogisms and enthymemes, which are types of argument. Lavenham is thus claiming that in an argument the understanding of the conclusion (consequent) necessarily belongs to the premisses (antecedent), which is a reasonable way to gloss Fland's criterion. The Lavenham-Fland account, then, is recognizably the same as our informal characterization of a valid argument as one in which the conclusion is "contained" in the premisses, and a cousin of the view that deductive inference is not ampliative—unlike, say, inductive inference—since the conclusion contains no more information than the premisses. Whether such an account can be made sufficiently precise for logical treatment is another matter. (Modern information-theoretic accounts of deducibility have not met with general acceptance.) Fland is alone among the authors in the first half of the fourteenth century in mentioning it, and so we shall set it aside for now.

The second and most common account of inferential validity among our authors is modal: the consequence $A \vdash B$ is legitimate when it is impossible for A to be true and B false. More precisely, the modal criterion spells out at least a necessary condition for consequences in general to be legitimate.[39] The intuition at work here is familiar. Modal accounts of logical consequence date back to Aristotle, and live on today in Tarskian model-theoretic explanations of logical consequence that take possible worlds to

[39] Mediaeval logicians drew several distinctions among kinds of consequences, such as the distinction between consequence *simpliciter* and consequence *ut nunc*. Does the modal account of validity range over times or just possibilities at a time? I'm inclined to the latter, and that the common mediæval view was that "all consequences are necessary" (as the *De consequentiis* possibly written by Bradwardine asserts in §7). I will proceed as though the question were settled, but it deserves more attention than I can give it here.

be the models in which an interpretation is evaluated.[40] Well, with Aristotle on one side and Tarski on the other, how did the mediæval logicians of the first half of the fourteenth century explicate the modal account of consequence?

A battery of distinctions were available that would allow them to construct a fairly precise nonmathematical analogue of Tarskian satisfaction. Jean Buridan offers a clear and lucid presentation of the material, so I'll concentrate on his exposition. Roughly, a sentence is true for Buridan when what it says is the case. (This claim has to be tweaked for tense and quality, of course, but we can ignore such niceties for now.) Thus a consequence $A \vdash B$ is legitimate when it is impossible for what A says to be the case and for what B says not to be the case. More precisely, it is impossible for the situation that B describes not to hold in the situation that A describes. These situations may be alternative possibilities. Buridan distinguishes between situations that a sentence may describe and also belong to, and those situations which it may describe but not belong to. This is his well-known distinction between sentences that are possibly-true and those that are (merely) possible.[41] For instance, the sentence "No sentence is negative" is possible but not possibly-true, because it describes a possible situation but cannot belong to it. Hence we can clearly distinguish a sentence from the situation it describes and also from its truth-value with respect to that situation. In modern terminology, a possible situation functions as a model, and sentences are assigned truth-values relative to the model. Such an assignment of truth-values is a nonmathematical version of Tarskian satisfaction. Hence the consequence $A \vdash B$ is legitimate when it is impossible for A to be true and B false, that is, when there is no situation in which A is assigned truth and B falsity. Inferential legitimacy is a function of the truth-value of sentences with respect to situations.

The situations are possibilities—possible worlds, if you like. They can be constructed to evaluate sentences, and were extensively used to do exactly that, particularly in the case of sophisms, where they supplied a technique for both modelling and countermodelling: the description of a situation (*casus*) was the starting-point of these investigations. Hence even if the fine

[40] The success of the model-theoretic notion of logical consequence, derived from (but not identical to) the version presented in Tarski [1935], has been challenged recently in Etchemendy [1990]. The discussion of these matters in Shapiro [1998] is extremely helpful, and I make use of his distinctions among accounts of logical consequence in what follows.

[41] Prior [1969] is the *locus classicus*. The distinction is drawn from the first two chapters of Buridan's *Sophismata* 8.

points of Buridan's account were not available to or accepted by everyone, the common use of possible situations in sophisms and obligations shows that the philosophical machinery for explicating the modal account of validity was widely available. The widespread agreement among mediæval logicians on the modal account didn't settle all the philosophical questions, however. Is quantification over such possibilities, as the modal account seems to demand, itself a legitimate procedure? What about inference from the impossible, where by definition there is no possible situation to start with? But put these difficulties, as challenging as they are, aside for the moment. There is a deeper worry about the proto-Tarskian theory sketched here, one recognized in the first half of the fourteenth century. As it stands, the account of truth (as a satisfaction-relation relative to a model) incorporated in the modal account has no clear connection with formal validity—or even with semantics at all.[42] How does inferential legitimacy depend on the formal features of sentences or on their meanings? Consider the three proposals that $A \vdash B$ is legitimate when:[43]

(1) The truth of A guarantees the truth of B in virtue of the meanings of the terms in each
(2) The truth of A guarantees the truth of B in virtue of the forms of A and B
(3) There is no uniform substitution of nonlogical terminology that renders A true and B false

Now to the extent that the meanings of the terms in A and B determine the situations—the range of possibilities or models—we evaluate our sentences against, (1) may provide a semantic dimension to the modal account. Yet (1) will fail to capture formal validity to the extent that meaning is not a formal feature.[44] Inferences such as "Socrates is human; therefore, Socrates is an animal" are legitimate by (1) but are not formally valid: they do not hold in virtue of their form but only hold in virtue of some extrinsic feature, such as the meanings of their terms or the way the world is. (Thus even metaphysical necessity does not entail formal validity.) Several mediæval logicians turned to the theory of topics to explain such materially

[42] This point can be pressed against Tarski, and is vigorously argued in Etchemendy [1990].

[43] Taken from Shapiro [1997] 132: his (6), (9), and (8) respectively.

[44] Modern logicians have made various attempts to treat meaning as a formal feature: see n. 35 above. Mediaeval logicians (made no such comparable attempt, although they were divided on how to treat certain kinds of structured meaning-relations (notably between subordinate and superordinate elements in a categorial line). One technique was to use the theory of topics—see the following note.

valid inferences, sometimes reducing them to formal ones, sometimes the converse.[45]

In contrast, the account of legitimacy proposed in (2) tries to explain it by connecting truth and formality. It can even be seen as a special case of (1), wherein the meanings of a special set of terms—called nowadays the 'logical vocabulary'—constitute the form of a given sentence. Of course, it isn't clear whether logical vocabulary has meaning at all, even when taken in combination with other terms (as the mediæval account of 'syncategorematic' terms presupposes). But even so there are three further problems with (2) as an explication of legitimacy. First, the very move from (1) to (2) is suspect. Why should we be interested in logical form in the first place? Why not be content with guaranteed truth, for which (1) is sufficient? It should be an open question whether all validity is formal validity, but (2) closes the subject. Second, mediæval and modern logicians alike have yet to come up with a criterion to identify the "form" of a sentence that doesn't simply beg the question. What is the logical form of a definite description? Of a paradoxical liar-like sentence? Of sentences involving the word "begins"? Third, even if we could specify the form of a sentence without begging any questions, we need to know how formal features determine possibilities.[46] For example, suppose that the (logical) form of the sentence "Socrates is older than Plato" is "x is older than y." Surely not all situations in which one thing is, or is claimed to be, older than another count as possibilities relevant to evaluating the original sentence. What bearing does the situation in which my piano is older than my violin have on Socrates's being older than Plato? It is not that the net of possibility is cast too widely—instead, it seems to be miscast. The age of my musical instruments is simply irrelevant to the respective ages of Socrates and Plato. Insofar as such possibilities are prescribed by (2), the intuitive punch of the modal account is lost.

Rather than taking possibilities to be spelled out by the meanings of terms of a sentence or by the structure of a sentence, we could instead look directly at the truth-value of sentences generated by altering a given sentences's (nonlogical) terminology. This is the key intuition behind the

[45] See Green-Pedersen [1984] and Stump [1989] for discussion of the use of topics in this period. (Interestingly, Tarski also speaks of "material consequences": Shapiro [1998] 148.) Burleigh, for example, says in the longer version of his *De puritate artis logicae* that every consequence holds in virtue of a logical topic (75.35–76.10). Ockham's awkward doctrine of intrinsic and extrinsic middles may be seen as addressing to some of these worries.

[46] See Shapiro [1998] 143 on interpretational and representational semantics.

substitutional account of legitimacy presented in (3). We can best judge legitimacy by seeing whether an inference holds in terms other than those in which it is originally couched. (Our ability to judge the truth-value of the candidate sentences is assumed.) Furthermore, to the extent that we can identify some terms as part of the logical vocabulary and so as elements of the form of the sentence, (3) will be a formal account as well. Hence Buridan, for example, endorses (3) as the correct account of legitimacy, specifically linking it to formality.[47] Uniform subsitutivity, of the sort proposed by Buridan, is the third account of inferential validity. He is clear that (3) goes beyond the modal account in at least two ways. First, it applies equally to material (non-formal) consequences; thus the formality of an inference is a feature that goes beyond its necessity, neither explained by nor reducible to it. Second, it takes legitimacy to be a function of truth-value relative to a set of terms, namely the nonlogical vocabulary, rather than appealing to possibilities. The third account of inferential validity therefore takes a decidedly linguistic and non-metaphysical approach.

The mediæval consensus on a proto-Tarskian account of satisfaction, then, conceals deep divergences in the attempt to explain legitimacy. It may be worthwhile to change direction in pursuing this problem. Rather than looking more closely at the nature of formal and material inference (a topic on which there doesn't seem to be much agreement among mediæval logicians), we can try to make some headway by understanding how mediæval logicians reasoned about possible situations and alternatives. Since consequences license arguments, where such possibilities are found, we can start there; after examining the nature of arguments I'll conclude with some reflections on formality and the logical enterprise.

5. Argument and Argumentation

Let me pick up a thread from §2 and return to the relation between conditionals and consequences. They are logically distinct notions. Are they correlated in any way? At the beginning of our period the question seems to be ignored, but by the end two schools of thought have emerged. On the one hand, Ockham and his followers hold that conditionals and consequences are logically interchangeable. In *Summa logicae* 2.31, Ockham

[47] *Tractatus de consequentiis* 22.5–9. See also *Summulae de dialectica* 1.6.1: "Et quia nunc locutum est de consequentia formali et materiali, uidendum est quo modo conueniant et differant: conueniunt enim in hoc quod impossibile est antecedens esse uerum consequente exsistente falso; sed differunt quia consequentia 'formalis' uocatur quae si ex quibuscumque terminis formarentur, propositiones similis formae ualeret similiter consequentia." Buridan's account of substitutivity is similar to Bolzano [1837].

declares that since a conditional is equivalent (*aequiualet*) to a consequence he'll just talk about the latter (347.2–5). So too the *Logica ad rudium* 2.76. The treatise *De consequentiis* §7 gives us the other direction: every legitimate consequence is equivalent to a true conditional (Green-Pedersen [1982] 93). The moral is eventually drawn in the fifteenth rule of the *Liber consequentiarum* 2:[48]

> Every consequence is equivalent to a conditional composed of the antecedent and consequent of the given consequence with 'if' put in front of the antecedent, and conversely every conditional is also equivalent to a consequence composed of the antecedent and consequent of the given conditional with 'therefore' put in front of the consequent.

This 'mediæval deduction theorem' permits the logician to pass between conditional and inferential formulations of the same claim, without any logical baggage getting lost in the transfer, as it were.

On the other hand, conditionals and consequences might be thought to differ in ways that prevent them from being simply exchanged for one another. This is the position of Jean Buridan (and Albert of Saxony, as usual, in his wake). He sketches the difference in his *Summulae de dialectica* 7.4.5, when he disentangles the sloppy use of 'consequence' in place of 'conditional':[49]

> Note that 'consequence' is twofold: (1) a conditional sentence, which asserts neither its antecedent nor its consequent (*e. g.* "if an ass flies an ass has wings") but only asserts that the one follows from the other, and such a consequence is not an argument since it doesn't prove anything; (2) an argument wherein the antecedent is known and better-known than the consequent, which asserts the antecedent and on that basis implies the consequent as an assertion. Furthermore, in a conditional we use 'if' and in an argument 'therefore'.

[48] Schupp [1988] 123.198–203: "Decima quinta regula est haec quod omnis consequentia aequiualet condicionali compositae ex antecedente et consequente illius consequentiae cum nota condicionalis praeposita antecedenti, et econuerso omnis condicionalis etiam consequentiae compositae ex antecedente et consequente illius condicionalis cum nota consequentiae praeposita consequenti."

[49] "Deinde notat duplicem esse consequentiam, scilicet unam quae est propositio condicionalis, et illa nec asserit antecedens nec asserit consequens (ut 'si asinus uolat, asinus habet alas'), sed solum asserit quod hoc sequitur ad illud. Et ideo talis consequentia non est argumentum; nihil enim concludit. Alia consequentia est argumentum si antecedens sit notum et notius consequente, quae asserit antecedens et ob hoc infert assertiue consequens. In condicionali autem utimur hac coniunctione 'si' et in argumento hac coniunctione 'ergo'."

Conditionals do not involve the assertion of their parts, whereas consequences do. Buridan makes the same point earlier as regards syllogisms (*Summulae de dialectica* 1.7.3):[50]

> The syllogism differs from the conditional sentence too, because in the conditional sentence its categorical parts aren't put forward in the manner of an assertion (*i. e.* affirmatively), whereas they are put forward in the manner of an assertion in syllogisms—*e. g.* that every B is A and every C is B, and the conclusion that every C is A is drawn in the manner of an assertion. Thus we say that a syllogism with false premisses is materially defective, which shouldn't be said of the conditional "If an ass is flying, an ass has wings."

An argument, as noted in §1, does not make a statement. Yet it does license the making of a statement by anyone who accepts its premisses, namely the detachable statement of its conclusion. Implication does not work like this: conditionals do make statements, namely statements about the relation between the antecedent and the consequent, but unlike arguments they do not license further statements. Hence, for Buridan, consequences and conditionals are not interchangeable.[51]

Buridan's view is that accepting or rejecting premisses, committing oneself to an inference, warranting further freestanding statements, and other activities that we might broadly call "dialectical" are partially constitutive of the sense of an argument.[52] However, we don't have to believe that giving an argument will automatically commit us to asserting its conclusion in order to take Buridan's point. Even Buridan didn't think so—otherwise, we wouldn't be able to draw conclusions from an opponent's views to refute

[50] "Et etiam syllogismus differt a propositione condicionali, quia in propositione condicionali nullo modo categoricae proponuntur modo assertiuo, id est affirmatiuo, sed in syllogismis proponuntur modo assertiuo, ut quod omne B est A et omne C est B, et concluditur assertiue quod omne C est A. Et ideo dicimus syllogismus ex falsis praemissis peccare in materia, quod non sic est dicendum de ista condicionali 'si asinus uolat, asinus habet pennas'."

[51] Modern logicians are divided over whether arguments do in fact license freestanding occurrences of their conclusions. For instance, if we think of logical consequence along purely syntactic lines, the formula $A \vdash B$ says only that B can be deduced from A—a claim that seems to carry no commitment to B (or presuppose any endorsement of A). This is evidence that Buridan and other mediæval logicians did not think of consequence as simple deducibility.

[52] Perhaps a more fine-grained approach would be useful here. David Kaplan's distinction of propositional context, character, and content allows us to state Buridan's claim more exactly: dialectical activities fix the character of propositions as they occur in arguments; they do not enter into their content.

him. Instead, the point well-taken from Buridan's discussion is that making an argument necessarily involves taking a dialectical stance. What an agent is doing dialectically in stringing together statements will depend, at least in part, on whether the agent is (say) accepting the statements, or rejecting them, or granting them temporarily, or is in doubt about what to do. Which commitments the agent has will depend on which dialectical stance he adopts.

The factors listed here as making up an agent's dialectical stance are, of course, precisely those that enter into *obligationes*. They enable arguments to be what they are in the first place, namely a kind of activity in which we do things with statements. And, as such, they are ways of doing things with statements.[53] For arguments are not independent objects that can be analyzed apart from the contexts in which they occur. Part of their sense—or at least part of what it is to string statements together in an inference-making performance—is arguably constituted by these obligational attitudes (for want of a better description).[54] Obligational treatises are, among other things explored at length in this volume, conscious attempts to work out how certain obligational attitudes are related to inferences. They are efforts to explore the logical features of arguments—dialectical performances—found in the wild.

To be sure, we can domesticate obligational attitudes to some extent. If we consider arguments solely from the point of view of accepted (or perhaps even conceded) premisses, a theory of valid consequence that makes only tacit reference to its dialectical origins can be constructed. This is, in essence, the theory of the syllogism. But the task of the mediæval logician is to examine arguments wherever they may be found, including their natural habitats, and on that reading obligations are part of logical theory proper. Yet obligational attitudes are not, or not in any obvious way, formal features of arguments—that is, they aren't part of the logical form of an argument as such; we seem to be able to talk about arguments without referring to their dialectical contexts. To get straight on how obligations are part of the logical enterprise, we need to look a bit more closely at formality and its

[53] Some of the dialectical features described here have been explored at length in Brandom [1994]. But his project of "inferential semantics" is not the mediæval one: to insist that arguments have some irreducible social features is a far cry from maintaining that (all) meaning is constituted by inferential roles and permissible moves of our language-games.

[54] If arguments depend on obligational attitudes for their sense, it is misleading to represent them as operators that extend or enrich an independent logical system—as, for example, in Boh [1993] (for epistemic operators).

connection with logic, both mediæval and modern. In so doing we'll get a better idea of the logical enterprise generally.

Conclusion

Modern logicians, who spend much of their time either devising logical systems that are mathematically-defined objects or investigating the properties of such systems (metatheory), are engaged in a fundamentally modern enterprise. Mediaeval logicians were in no position to do either of these tasks. Yet mediæval logic is still logic, after all; its relation to modern logic is not like the relation of alchemy to chemistry. The glory of modern logic is rather that it succeeds in treating logic mathematically. But logic is not intrinsically mathematical; it would have little past before *Principia mathematica* if it were. Yet the influence of mathematics on logic has undeniably changed its character: mediæval and modern logic are overlapping but distinct enterprises. Each is concerned with logic as in some sense the study of correct reasoning, but without more content this slogan doesn't get us very far. What more can be said? Well, mediæval and modern logic both attempt to be rigorous and systematic. And, more importantly, each is a formal discipline. That is, they are concerned with studying properties of formal features, *e. g.* determining which inferences hold in virtue of the logical form of the premisses and of the conclusion (truth-preserving formal inferences). Modern logic is formal and formalized (symbolic); mediæval logic is formal but not formalized. To this extent Ockham and Tarski are engaged in a common endeavor and the history of logic stretches back to Aristotle.

Mediaeval logic is also nonformal.[55] That is, mediæval logic deals with inferences and assertions that do not hold in virtue of their formal features as well as those that do. Here Ockham and Tarski part ways: modern logic concentrates exclusively on formal properties whereas mediæval logic is more inclusive. Some sense of the scope of mediæval logic can be gotten by looking at the variety of subjects falling within its scope: semantics, reference, syncategoremata, syllogistic, consequences, topics, sophisms, paradoxes, obligations, and fallacies. Yet I think there is a single conception of logic here, with consequences at its heart. It is this. Mediaeval logic is the enterprise of devising theories about inference. Inferences may be formal

[55] This is not the same as our modern conception of informal logic, which is at best the general study of deductive and inductive reasoning, the latter based on probability and statistics. Unfortunately, "informal logic" is usually taken to be synonymous with "critical thinking": equal parts of rhetoric, traditional fallacies, and epistemic good sense. There is nothing particularly logical about informal logic taken in this sense.

or material, legitimate or illegitimate, and are found in different dialectical circumstances. The unity of mediæval logic is grounded in its conception of inference (consequence), the key to nonformal logic. Now mediæval logic is recognizably related to modern mathematical logic, since it studies formal legitimate inferences, the sole subject of modern logic. But it also studies much else besides, such as illegitimate inferences (the theory of fallacies).

Whether the mediæval conception of logic as the nonformal study of inference is a worthy competitor to the modern mathematical conception of logic is another question. We cannot make a start on answering it until we recognize the centrality of the notion of inference in mediæval logic. An obvious first step in that process is clarifying the notion of inference itself. As I have argued here, this was accomplished in the first half of the fourteenth century through a natural deduction system and articulated in discussions of consequences, which are the heart of argument and, by extension, the very heart of (mediæval) logic itself.[56]

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Bibliography

Mediæval Texts 1300–1350

Anonymous. *Anonymi de consequentiis* (ca. 1300). Edited by Niels Jørgen Green-Pedersen in "Two Early Anonymous Tracts on Consequences". *Cahiers de l'institut du moyen-âge grec et latin* 35 (1980) 1–28 (text 4–11).

Anonymous. *Anonymi de consequentiis* (ca. 1300). Edited by Niels Jørgen Green-Pedersen in "Two Early Anonymous Tracts on Consequences". *Cahiers de l'institut du moyen-âge grec et latin* 35 (1980) 1–28 (text 12–28).

Anonymous. *Liber consequentiarum* (1330–1340). Edited in Schupp [1988] 104–171.

Anonymous. *De consequentiis* (1325–1340). Edited by Niels Jørgen Green-Pedersen in "Bradwardine(?) On Ockham's Doctrine of Consequences: An Edition." *Cahiers de l'institut du moyen-âge grec et latin* 42 (1982) 85–150.

Anonymous. *Logica "Ad Rudium"* (1335). Edited by L. M. De Rijk. *Artistarium* 1. Ingenium Publishers: Nijmegen 1981.

Buridan, Jean. *Tractatus de consequentiis* (1335). Edited by Hubert Hubien in *Iohannis Buridani tractatus de consequentiis. Philosophes médiévaux* 16. Publications Universitaires: Louvain 1976. Translated in King [1985].

Buridan, Jean. *Summulae de dialectica* (1340s?). References are to book, chapter, and section number. The second book has been edited by L. M. De Rijk as the *Summulae de praedicabilibus* (Nijmegen: Ingenium 1995). The third book has been edited by E. P. Bos as the *Summulae in Praedicamenta* (Nijmegen: Ingenium 1994). For other parts of the text I have used the unpublished transcription of Hubert Hubien, employed in the critical editions, and provide the Latin in the notes.

Burleigh, Walter. *De consequentiis* (1300). Edited by Niels Jørgen Green-Pedersen in "Walter Burley's 'De Consequentiis': An Edition." *Franciscan Studies* n.s. 40 (1980) 102-166.

Burleigh, Walter. *De puritate artis logicae (tractatus brevior)* (early 1320s). Edited by Philotheus Boehner O.F.M. in *Walter Burleigh: De puritate artis logicae, Tractatus Longior. With a Revised Edition of the Tractatus Brevior.* The Franciscan Institute: St Bonaventure 1955, 199–260.

Burleigh, Walter. *De puritate artis logicae (tractatus longior)* (later 1320s). Edited by Philotheus Boehner O.F.M. in *Walter Burleigh: De puritate artis logicae, Tractatus Longior. With a Revised Edition of the Tractatus Brevior.* The Franciscan Institute: St Bonaventure 1955, 1–197.

Ferrybridge, Richard. *Consequentiae* (ca. 1350). Printed in *Consequentiae Strodi cum commento Alexandri Sermonetae. Declarationes Gaetani in*

easdem consequentias. Dubia magistri Pauli Pergulensis. Obligationes eiusdem Strodi. Consequentiae Ricardi de Ferabrich. Expositio Gaetani super easdem. Venetiis 1507.

Fland, Robert. *Consequentiae* (1350s?). Edited by Paul Spade in "Robert Fland's Consequentiae: An Edition." *Mediaeval Studies* 38 (1976) 54–84.

Pseudo-Ockham. *Tractatus minor logicae* (ca. 1340–1347). Originally edited by Eligius M. Buytaert; revised by Gedeon Gál and Joachim Giermek. Printed in William of Ockham, *Opera philosophica* 7 (opera dubia et spuria). The Franciscan Institute: St Bonaventure 1988, 1–57.

Pseudo-Ockham. *Elementarium logicae* (ca. 1340–1347). Originally edited by Eligius M. Buytaert; revised by Gedeon Gál and Joachim Giermek. Printed in William of Ockham, *Opera philosophica* 7 (opera dubia et spuria). The Franciscan Institute: St Bonaventure 1988, 58–304.

Pseudo-Scotus. *Super librum primum et secundum Priorum Analyticorum Aristotelis quaestiones* (around 1350). John of Cornwall? Printed and wrongly ascribed to Scotus in *Joannis Duns Scoti Doctoris Subtilis Ordinis Minorum opera omnia*, ed. Luke Wadding, Lyon 1639; republished, with only slight alterations, by L. Vivès, Paris 1891–1895. Question 10 is translated in the present volume.

William of Ockham. *Summa logicae* (1323). Edited by Philotheus Boehner O.F.M., Gedeon Gál O.F.M., Stephanus Brown. *Opera philosophica* 1. The Franciscan Institute: St Bonaventure 1974.

Selected Mediæval Texts 1350–1400

Albert of Saxony. *Perutilis logica* (1370s). Venetiis 1522. Reprinted in *Documenta Semiotica* 6. Georg Olms: Hildesheim 1974.

Lavenham, Richard. *Consequentiae* (1370). Edited by Paul Spade in "Five Logical Tracts by Richard Lavenham" in *Essays in Honor of Anton Charles Pegis*, ed. J. R. O'Donnell. University of Toronto Press 1974, 70–124 (text 99–112).

Strode, Ralph. *Tractatus de consequentiis* (1360?). Text and translation in W. K. Seaton, *An Edition and Translation of the Tractatus de consequentiis of Ralph Strode*. Ph.D. Dissertation, University of California at Berkeley 1973. University Microfilms: Ann Arbor 1974.

Modern Works

Adams [1987]. Marilyn Adams, *William Ockham*. 2 volumes. University of Notre Dame Press 1987.

Boh [1962]. Ivan Boh, "A Study in Burleigh: *Tractatus de regulis generalibus consequentiarum*" in *The Notre Dame Journal of Formal Logic* 3 (1962) 83–101.

Boh [1982]. Ivan Boh, "Consequences" in *The Cambridge History of Later Medieval Philosophy*. Norman Kretzmann, Anthony Kenny, Jan Pinborg, eds. Cambridge University Press 1982.

Boh [1993]. Ivan Boh, *Epistemic Logic in the Later Middle Ages*. Routledge & Kegan Paul: London 1993.

Bolzano [1837]. Bernard Bolzano, *Wissenschaftslehre* (4 vols.), 1837.

Bos [1976]. E. P. Bos, "John Buridan and Marsilius of Inghen on Consequences" in Jan Pinborg, ed. *The Logic of John Buridan*. Acts of the Third European Symposium on Medieval Logic and Semantics. Museum Tusculanum: Copenhagen 1976.

Brandom [1994]. Robert B. Brandom, *Making It Explicit. Reasoning, Representing, and Discursive Commitment*. Harvard University Press 1994.

Carroll [1895]. Lewis Carroll, "What the Tortoise Said to Achilles" in *Mind* 4 (1895) 278–280.

Corcoran [1974]. John Corcoran, "Aristotle's Natural Deduction System" in *Ancient Logic and Its Modern Interpretations*, ed. John Corcoran. D. Reidel: Dordrecht 1973.

Dummett [1973]. Michael Dummett, "The Justification of Deduction" in *Proceedings of the British Academy* 1973 201–232.

Etchemendy [1990]. John Etchemendy, *The Concept of Logical Consequence*. Harvard University Press 1990.

Gentzen [1935]. Gerhardt Gentzen, "Untersuchungen über das logische Schliessen" in *Mathematische Zeitschrift* 39 (1935) 176–210 and 405–431.

Green-Pedersen [1983]. Niels Jørgen Green-Pedersen, "Early British Treatises on Consequences" in *The Rise of British Logic*. Acts of the Sixth European Symposium on Medieval Logic and Semantics. Edited by P. Osmund Lewry, O.P. Papers in Mediaeval Studies 7. Pontifical Institute of Mediaeval Studies: Toronto 1983 285–307.

Green-Pedersen [1984]. Niels Jørgen Green-Pedersen, *The Tradition of the Topics in the Middle Ages*. Philosophia Verlag: München 1984.

Haack [1976]. Susan Haack, "The Justification of Deduction" in *Mind* 95 (1982) 216–239.

Jackson [1991]. *Conditionals*. Edited by Frank Jackson. Oxford University Press 1991.

Jaskowski [1934]. Stanislaw Jaskowski, "On the Rules of Suppositions in Formal Logic" in *Studia logica* 1 (1934), 5–32.

King [1985]. Peter King, *Jean Buridan's Logic: The Treatise on Supposition and the Treatise on Consequences*. D. Reidel: Dordrecht 1985.

Kneale & Kneale [1962]. William Kneale and Martha Kneale, *The Development of Logic*. Oxford University Press 1962.

Lear [1980]. Jonathan Lear, *Aristotle and Logical Theory*. Cambridge University Press 1980.

McCall [1966]. Storrs McCall, "Connexive Implication" in *The Journal of Symbolic Logic* 31 (1966) 415–433.

Moody [1953]. Ernest A. Moody, *Truth and Consequence in Medieval Logic*. North-Holland: Amsterdam 1953.

Nute [1980]. Donald Nute, *Topics in Conditional Logic*. D. Reidel: Dordrecht 1980.

Pozzi [1978]. Lorenzo Pozzi, *Le consequentiae nella logica medievale*. Liviana editrice: Padova 1978.

Pinborg [1972]. Jan Pinborg, *Logik und Semantik im Mittelalter*. Ein Überblick. Fromann-Holzboog: Stuttgart 1972.

Prawitz [1965]. Dag Prawitz, *Natural Deduction: A Proof-Theoretical Study* Almqvist & Wiksell: Stockholm 1965.

Prior [1953]. Arthur Prior, "On Some Consequentiae in Walter Burleigh" in *New Scholasticism* 27 (1953) 433–446.

Prior [1969]. Arthur Prior, "The Possibly-True and the Possible" in *Mind* 78 (1969) 481–492.

Schupp [1981]. Franz Schupp, *Logical Problems of the Medieval Theory of Consequences*. History of Logic 6. Bibliopolis: Napoli 1988.

Shapiro [1998]. Stewart D. Shapiro, "Logical Consequence: Models and Modality" in *Philosophy of Mathematics Today*, ed. Mathias Schirn. Oxford University Press 1998, 131–156.

Smiley [1973]. Timothy Smiley, "What is a Syllogism?" in *The Journal of Philosophical Logic* 2 (1973) 136–154.

Smith [1989]. Robin Smith, *Aristotle: Prior Analytics*. Hackett Publishing Company: Indianapolis 1989.

Stump [1989]. Eleonore Stump, *Dialectic and Its Place in the Development of Medieval Logic*. Cornell University Press 1989.

Tarski [1935]. Alfred Tarski, "Der Wahrheitsbegriff in den formalisierten Sprachen" in *Studia philosophica* 1 (1935), 261–405. (Derived from an earlier Polish paper published in 1933.)

Thom [1981]. Paul Thom, *The Syllogism*. Philosophia Verlag: München 1981.

Woods [1997]. Michael Woods, *Conditionals*. Edited by David Wiggins, with a commentary by Dorothy Edgington. Clarendon Press: Oxford 1997.