THE HISTORY OF LOGIC

Aristotle was the first thinker to devise a logical system. He drew upon the emphasis on universal definition found in Socrates, the use of *reductio ad absurdum* in Zeno of Elea, claims about propositional structure and negation in Parmenides and Plato, and the body of argumentative techniques found in legal reasoning and geometrical proof. Yet the theory presented in Aristotle's five treatises known as the *Organon*—the *Categories*, the *De interpretatione*, the *Prior Analytics*, the *Posterior Analytics*, and the *Sophistical Refutations*—goes far beyond any of these.

Aristotle holds that a proposition is a complex involving two terms, a subject and a predicate, each of which is represented grammatically with a noun. The logical form of a proposition is determined by its quantity (universal or particular) and by its quality (affirmative or negative). Aristotle investigates the relation between two propositions containing the same terms in his theories of opposition and conversion. The former describes relations of contradictoriness and contrariety, the latter equipollences and entailments.

The analysis of logical form, opposition, and conversion are combined in syllogistic, Aristotle's greatest invention in logic. A syllogism consists of three propositions. The first two, the premisses, share exactly one term, and they logically entail the third proposition, the conclusion, which contains the two non-shared terms of the premisses. The term common to the two premisses may occur as subject in one and predicate in the other (called the 'first figure'), predicate in both ('second figure'), or subject in both ('third figure'). A given configuration of premisses and conclusions is called a 'mood'.

In the scholastic period, mnemonic names for the valid moods canvassed in the *Prior Analytics* were devised. Two first-figure valid moods were considered perfect and not in need of any further validation: BARBARA (consisting entirely in universal affirmatives) and CELARENT (consisting in a universal negative and a universal affirmative, concluding in a universal negative). For the validation of the rest, Aristotle used three techniques: reduction, in which a given mood is transformed through conversions into BARBARA or CELARENT; *reductio ad absurdum*; and ἔκθεσις, which proceeds by selection of an arbitrary individual. He regularly describes moods by using variables in place of terms. To reject a proposed inference he typically gives a list of terms that, when substituted as values of the term-variables, produce true premisses with false conclusions. This is similar to the modern

technique of constructing counterarguments to establish invalidity.

Aristotle may also be credited with the formulation of several metalogical theses, most notably the Law of Noncontradiction, the Principle of the Excluded Middle, and the Law of Bivalence. These are important in his discussion of modal logic and tense logic. Aristotle referred to certain principles of propositional logic and to reasoning involving hypothetical propositions. He also created to non-formal logical theories: techniques and strateies for devising arguments (in the *Topics*), and a theory of fallacies (in the *Sophistical Refutations*). Aristotle's pupils Eudemus and Theophrastus modified and developed Aristotelian logic in several ways.

The next major innovations in logic are due to the Megarian-Stoic School. They developed an alternative account of the syllogism, and, in the course of so doing, elaborated a full propositional logic which complements Aristotelian term logic. There are fragmentary records of debates over the truth-conditions for various propositional connectives, which include accounts of material implication, strict implication, and relevant implication. The Megarians and the Stoics also investigated various logical antinomies, including the Liar Paradox. The leading logician of this school was Chrysippus, credited with over a hundred works in logic.

There were few developments in logic in the succeeding periods, other than a number of handbooks, summaries, translations, and commentaries, usually in a simplified and combined form. The more influential authors include Cicero, Porphyry, and Boethius in the later Roman Empire; the Byzantine scholiast Philoponous; and al-Farabi, Avicenna, and Averroës in the Arab world.

The next major logician known to us is an innovator of the first rank: Peter Abelard, who worked in the early twelfth century. He composed an independent treatise on logic, the *Dialectica*, and wrote extensive commentaries. There are discussions of conversion, opposition, quantity, quality, tense logic, a reduction of *de dicto* to *de re* modality, and much else. Abelard also clearly formulates several semantic principles, including the Tarski biconditional for the theory of truth, which he rejects. Perhaps most important, Abelard is responsible for the clear formulation of a pair of relevant criteria for logical consequences. The failure of his criteria led later logicians to reject relevance implication and to endorse material implication.

Spurred by Abelard's teachings and problems he proposed, and by further translations, other logicians began to grasp the details of Aristotle's texts. The result, coming to fruition in the middle of the thirteenth century, was the first phase of supposition theory, an elaborate doctrine about the reference of terms in various propositional contexts. Its development is pre-

served in handbooks by Peter of Spain, Lambert of Auxerre, and William of Sherwood. The theory of *obligationes*, a part of non-formal logic, was also invented at this time. Other topics, such as the relation between time and modality, the conventionality of semantics, and the theory of truth, were investigated.

The fourteenth century is the apex of mediæval logical theory, containng an explosion of creative work. Supposition theory is developed extensively in its second phase by logicians such as William of Ockham, Jean Buridan, Gregory of Rimini, and Albert of Saxony. Buridan also elaborates a full theory of consequences, a cross between entailments and inference rules. From explicit semantic principles, Buridan constructs a detailed and extensive investigation of syllogistic, and offers completeness proofs. Nor is Buridan an isolated figure. Three new literary genres emerged: treatises on syncategoremata (logical particles), which attempted to codify their behaviour and the inferences they license; treatises on sentences called 'sophisms' that are puzzling or challenging given background assumptions about logic and language; and treatises on insolubles, such as the Liar Paradox.

The creative energy that drove the logical inquiries of the fourteenth century was not sustained. By the middle of the fifteenth century little if any new work was being done. There were instead many simplified handbooks and manuals of logic. The descendants of these textbooks came to be used in the universities, and the great innovations of mediæval logicians were forgotten. Probably the best of these works is the *Port Royal Logic*, by Antoine Arnauld and Pierre Nicole, published in 1662. When writers refer to 'traditional logic' they usually have this degenerate textbook tradition in mind.

Since the beginning of the modern era most of the contributions to logic have been made by mathematicians. Leibniz envisioned the development of a universal language to be specified with mathematical precision. The syntax of the words is to correspond to the metaphysical make-up of the designated entities. The goal, in effect, was to reduce scientific and philosophical speculation to computation. Although this grandiose project was not developed very far, and it did not enjoy much direct influence, the Universal Characteristic is a precursor to much of the subsequent work in mathematical logic.

In the early nineteenth century Bolzano developed a number of notions central to logic. Some of these, like analyticity and logical consequence, are seen to be relative to a collection of 'variable' concepts. For example, a proposition C is a consequence of a collection P of propositions relative to a group G of variable items, if every appropriate uniform substitution

for the members of *G* that makes every member of *P* true also makes *C* true. This may be the first attempt to characterize consequence in non-modal terms, and it is the start of a long tradition of characterizing logical notions in semantic terms, using a distinction between logical and non-logical terminology.

Toward the end of the nineteenth century one can distinguish three overlapping traditions in the development of logic. One of them originates with Boole and includes, among others, Peirce, Jevons, Schröder, and Venn. This 'algebraic school' focussed on the relationship between regularities in correct resaoning and operations like addition and multiplication. A primary aim was to develop calculi common to the reasoning in different areas, such as propositions, classes, and probabilities. The orientation is that of abstract algebra. One begins with one or more systems of related operations and articulates a common abstract structure. A set of axioms is then formulated which is satisfied by each of the systems. The system that Boole developed is quite similar to what is now called Boolean algebra. Other members of the school developed rudimentary quantifiers, which were sometimes taken to be extended, even infinitary, conjunctions and disjunctions.

The aim of the second tradition, the 'logicist school', was to codify the underlying logic of all reational scientific discourse into a single system. For them, logic is not the result of abstractions from the reasoning in particular disciplines and contexts. Rather, logic concerns the most general features of actual precise discourse, features independent of subject-matter.

The major logicists were Russell, the early Wittgenstein, perhaps, and the greatest logician since Aristotle, Gottlob Frege. In his *Begriffsschrift* (translated in van Heijenoort (ed.), *From Frege to Gödel*), Frege developed a rich formal language with mathematical rigour. Despite the two-dimensional notation, it is easily recognized as a contemporary higher-order logic. Quantifiers are understood as they are in current logic textbooks, not as extended conjunctions and disjunctions. Unlike the algebraists, Frege did not envision various domains of discourse, each of which can seve as an interpretation of the language. Rather, each (first-order) variable is to range over all objects whatsoever. Moreover, in contemporary terms, the systems of the logicists had no non-logical terminology.

Frege made brilliant use of his logical insights when developing his philosophical programmes concerning mathematics and language. He held that arithmetic and analysis are parts of logic, and made great strides in casting number theory within the system of the *Begriffsschrift*. To capture mathematical induction, minimal closures, and a host of other mathemat-

ical notions, he developed and exploited the ancestral relation, in purely logical terms.

Unfortunately, the system Frege eventually developed was shown to be inconsistent. It entails the existence of a concept R which holds of all and only those extensions that do not contain themselves. A contradiction known as "Russell's Paradox" follows.

A major response was the multi-volume *Principia mathematica* by Russell and Whitehead, which attempts to recapture the logicist programme by developing an elaborate theory of types. Antinomies are avoided by enforcing a vicious circle principle that no item may be defined by reference to a totality that contains the item to be defined. Despite its complexity, *Principia mathematica* enjoyed a wide influence among logicians and philosophers. An elegant version of the theory, called simple type theory, was introduced by Ramsey. It violates the vicious-circle principle, but still avoids formal paradox.

The third tradition dates back to at least Euclid and, in this period, includes Dedekind, Peano, Hilbert, Pasch, Veblen, Huntington, Heyting, and Zermelo. The aim of this 'mathematical school' is the axiomatization of particular branches of mathematics, like geometry, arithmetic, analysis, and set theory. Zermelo, for example, produced an axiomatization of set theory in 1908, drawing on insights of Cantor and others. The theory now known as Zermelo-Fraenkel set theory is the result of some modifications and clarifications, due to Skolem, Fraenkel, and von Neumann, among others.

Unlike Euclid, some members of the mathematical school thought it important to include an explicit formulation of the rules of inference—the logic—in the axiomatic development. In some cases, such as Hilbert and his followers, this was part of a formalist philosophical agenda, sometimes called the Hilbert programme. Others, like Heyting, produced axiomatic versions of the logic of intuitionism and intuitionistic mathematics, in order to contrast and highlight their revisionist programmes (see Brouwer).

A variation on the mathematical theme took place in Poland under Lukasiewicz and others. Logic itself became the branch of mathematics to be brought within axiomatic methodology. Systems of propositional logic, modal logic, tense logic, Boolean algebra, and mereology were designed and analysed.

A crucial development occurred when attention was focused on the languages and the axiomatizations themselves as objects for direct mathematical study. Drawing on the advent of non-Euclidean geometry, mathematicians in this school considered alternative interpretatins of their lan-

guages and, at the same time, began to consider metalogical questions about their systems, including issues of independence, consistency, categoricity, and completeness. Both the Polish school and those pursuing the Hilbert programme developed an extensive programme for such 'meta-mathematical' investigation. Eventually, notions about syntax and proof, such as consistency and derivability, were carefully distinguished from semantic or model-theoretic counterparts, such as satisfiability and logical consequence.

This metamathematical perspective is foreign to the logicist school. For them, the relevant languages were already fully interpreted, and were not going to be limited to any particular subject-matter. Because the languages are completely general, there is no interesting perspective 'outside' the system from which to study it. The orientation of the logicists has been called 'logic as language', and that of the mathematicians and algebraists 'logic as calculus'. Despite problems of communication, there was significant interaction between the schools. Contemporary logic is a blending of them.

In 1915 Löwenheim carefully delineated what would later be recognized as the first-order part of a logical system, and showed that if a first-order formula is satisfiable at all, then it is satisfiable in a countable (or finite) domain. He was firmly rooted in the algebraic school, using techiques developed there. Skolem went on the generalize that result in several ways, and to produce more enlightening proofs of them. The results are known as the Löwenheim-Skolem theorems.

The intensive work on metamathematical problems culminated in the achievements of Kurt Gödel, a logician whose significance ranks with Aristotle and Frege. In his 1929 doctoral dissertation, Gödel showed that a given first-order sentence is deducible in common deductive systems for logic if and only if it is logically true in the sense that it is satisfied by all interpretations. This is known as Gödel's completeness theorem. A year later, he proved that for common axiomatizations of a sufficiently rich version of arithmetic, there is a sentence which is neither provable nor refutable therein. This is called Gödel's incompleteness theorem, or simply Gödel's theorem.

The techniques of Gödel's theorem appeat to be general, applying to any reasonable axiomatization that includes a sufficient amount of arithmetic. But what is 'reasonable'? Intuitively, an axiomatization whould be effective: there should be an algorith to determine whether a given string is a formula, an axiom, *etc*. But what is an 'algorithm'? Questions like this were part of the motivatrion for logicians to turn their attention to the no-

tions of computability and effectiveness in the middle of the 1930s. There were a number of characterizations of cimplutability, developed more or less independently, by logicians like Gödel (recursiveness), Post, Church (lamba-definability), Kleene, Turing (the Turing machine), and Markov (the Markov algorithm). Many of these were by-products of other research in mathematical logic. It was shown that all of the characterizations are co-extensive, indicating that an important class had been identidied. Today, it is widely held that an arithmetic function is computable if and only if it is recursive, Turing machine computable, *etc*. This is known as Church's thesis.

Later in the decade Gödel developed the notion of set-theoretic constructibility, as part of his proof that the axiom of choice and Cantor's continuum hypothesis are consistent with Zermelo-Fraenkel set theory (formulated without the axiom of choice). In 1963 Paul Cohen showed that these statements are independent of Zermelo-Fraenkel set theory, introducing the powerful technique of forcing. There was (and is) a spirited inquiry among set theorists, logicians, and philosophers, including Gödel himself, into whether assertins like the continuum hypothesis have determinate truth-values.

Alfred Tarski, a pupil of Lukasiewicz, was one of the most creative and productive logicians of this, or any other, period. His influence spreads among a wide range of mathematical and philosophical schools and locations. Among philosophers, he is best known for his definitions of truth and logical consequence, which introduce the fruitful semantic notion of satisfaction. This, however, is but a small fraction of his work, which illuminates the methodology of deductive systems, and such central notions as completeness, decidability, consistency, satisfiablility, and definability. His results are the doundation of several ongoing research programmes.

Alonzo Church was another major influence in both mathematical and philosophical logic. He and students such as Kleene and Henkin have developed a wide range of areas in philosophical and mathematical logic, including completeness, definability, computability, and a number of Fregean themes, such as second-order logic and sense and reference. Church's theorem is that the collection of first-order logical truths is not recursive. It follows from this and Church's thesis that there is no algorithm for determining whether a given first-order formula is a logical truth. Church was a founder of the Association for Symbolic Logic and long-time guiding editor of the *Journal of Symbolic Logic*, which began publication in 1936. Volumes 1 and 3 contain an extensive bibliography of work in symbolic logic since antiquity.

The development of logic in the first few decades of the twentieth century is one of the most remarkable events in intellectual history, bringing together many brilliant minds working on closely related concepts.

Mathematical logic has come to be a central tool of contemporary analytic philosophy, forming the backbone of the work work of major figures like Quine, Kripke, Davidson, and Dummet. Since about the 1950s special topics of interest to contemporary philosophers, such as modal logic, tense logic, many-valued logic (used in the study of vagueness), deontic logic, relevance logic, and non-standard logic, have been vigourously studied. The field still attracts talented mathematicians and philosophers, and there is no sign of abatement.

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