FFT-Based Option Pricing under Mean-Reverting Lévy Processes

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1 Motivation

2 Mean-Reverting Fourier Space Time-stepping method
   - One-dimensional mean reversion with jump-diffusion
   - Multi-dimensional general framework

3 Numerical Results and Applications
   - Single-asset European, American and barrier options
   - Multi-asset spread options
   - Swing options

4 Conclusions
Commodities - Oil, Gas, Electricity,...

- Exhibit high volatilities and spikes in prices
- Tend to revert to long run equilibrium prices
- Many complex commodity contingent claims exist in the markets, such as swing and interruptible options
Henry Hub Natural Gas Prices
UK National Grid Electricity Prices

![Graph showing electricity prices from March 2004 to July 2008. The x-axis represents months from March 2004 to July 2008, and the y-axis represents System Sell Price (£/MWh) ranging from 0 to 200. The graph displays a trend of price fluctuations during this period.](image-url)
Existing Methods for Option Pricing

- Monte Carlo and tree methods
  - Slow convergence and expensive in computing the Greeks
- Finite difference methods
  - Integral term computationally expensive to handle; difficult to extend to multi-dimensional setting
- Early (fast) Fourier transform methods
  - Limited to European options; require Fourier transform of the payoff function
- Current fast Fourier transform (FFT) methods
  - Can only be applied to Lévy processes with independent increments
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The Option Pricing Problem

- Option payoff is given by $\varphi(S)$
- The commodity spot price $S_t = e^{X_t}$ is driven by mean-reverting jump-diffusion process

\[
dX_t = \kappa(\theta - X_t)dt + \sigma dW_t + dJ_t, \quad X_0 = \ln S_0
\]
The Option Pricing Problem

- Option payoff is given by $\varphi(S)$
- The commodity spot price $S_t = e^{X_t}$ is driven by mean-reverting jump-diffusion process

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t + dJ_t, \quad X_0 = \ln S_0$$

Option Value PIDE

$$\begin{cases} \partial_t v + \mathcal{L}v &= 0 \\ v(T, x) &= \varphi(e^x) \end{cases}$$

where $\mathcal{L}$ is the infinitesimal generator:

$$\mathcal{L}f(x) = \kappa(\theta - x)\partial_x f(x) + \frac{1}{2}\sigma^2 \partial_{xx} f(x) + \int (f(x + y) - f(x)) \nu(dy)$$
Let \( h_{t,x}(\omega, T) \triangleq \mathbb{E}^Q[e^{i\omega X_T} | \mathcal{F}_t] \) be the characteristic function of the log-stock price density under the risk-neutral measure.
Solving for the Characteristic Function

- Let $h_{t,x}(\omega, T) \triangleq \mathbb{E}^Q[e^{i\omega X_T} | \mathcal{F}_t]$ be the characteristic function of the log-stock price density under the risk-neutral measure.
- Assume ansatz form:

$$h_{t,x}(\omega, T) = e^{\psi^T_t(\omega) + \Phi^T_t(\omega)x}$$
Solving for the Characteristic Function

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- Assume ansatz form:
  \[
  h_{t,x}(\omega, T) = e^{\Psi^T_t(\omega) + \Phi^T_t(\omega)x}
  \]

- Since \( h_{t,x}(\omega) \) is a martingale, it satisfies the PIDE for all \( x \):
  \[
  (\partial_t + \mathcal{L}) h_{t,x} = \left( \dot{\Psi}^T_t + \dot{\Phi}^T_t x + \kappa(\theta - x)\Phi^T_t + \frac{1}{2} \sigma^2 (\Phi^T_t)^2 + \int (e^{\Phi^T_t y} - 1) \nu(dy) \right) h_{t,x} = 0
  \]
Solving for the Characteristic Function

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- Since $h_{t,x}(\omega)$ is a martingale, it satisfies the PIDE for all $x$:

$$(\partial_t + \mathcal{L}) h_{t,x} = (\dot{\Psi}^T_t + \dot{\Phi}^T_t x + \kappa(\theta - x)\Phi^T_t + \frac{1}{2}\sigma^2(\Phi^T_t)^2
+ \int(e^{\Phi^T_t y} - 1)\nu(dy)) h_{t,x} = 0$$

- $\Psi^T_t(\omega)$ and $\Phi^T_t(\omega)$ satisfy a system of Riccati ODEs:

$$\begin{cases}
\dot{\Psi}^T_t + \kappa \theta \Phi^T_t + \frac{1}{2}\sigma^2(\Phi^T_t)^2 + \int(e^{\Phi^T_t y} - 1)\nu(dy) = 0 \\
\dot{\Phi}^T_t - \kappa \Phi^T_t = 0,
\end{cases}$$

subject to $\Psi^T_T(\omega) = 0$ and $\Phi^T_T(\omega) = i\omega$.
We can solve for $\Phi_t^T(\omega)$ analytically:

$$\Phi_t^T(\omega) = i\omega e^{-\kappa(T-t)}$$
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$$\Phi_t^T(\omega) = i\omega e^{-\kappa(T-t)}$$

$\Psi_t^T(\omega)$ can then be solved:

$$\Psi_t^T(\omega) = \kappa \theta \int_t^T \Phi_s^T ds + \frac{1}{2} \sigma^2 \int_t^T (\Phi_s^T)^2 ds + \int_t^T \int_s^T (e^{\Phi_s^T y} - 1) \nu(dy) ds$$

$$= i\omega \theta (1 - e^{-\kappa(T-t)}) - \frac{\omega^2 \sigma^2}{4\kappa} (1 - e^{-2\kappa(T-t)}) + \int_t^T \tilde{\psi}(\omega e^{-\kappa(T-s)}) ds$$

where $\tilde{\psi}$ is the characteristic function of the jump distribution.
Solving for the Characteristic Function (cont.)

- We can solve for $\Phi_t^T(\omega)$ analytically:
  \[ \Phi_t^T(\omega) = i\omega e^{-\kappa(T-t)} \]

- $\Psi_t^T(\omega)$ can then be solved:
  \[
  \Psi_t^T(\omega) = \kappa\theta \int_t^T \Phi_s^T ds + \frac{1}{2} \sigma^2 \int_t^T (\Phi_s^T)^2 ds + \int_t^T \int_s^T (e^{\Phi_s^T y} - 1) \nu(\,d\,y)\,ds
  \]
  \[
  = i\omega\theta(1 - e^{-\kappa(T-t)}) - \frac{\omega^2\sigma^2}{4\kappa}(1 - e^{-2\kappa(T-t)}) + \int_t^T \tilde{\psi}(\omega e^{-k(T-s)})\,ds
  \]
  where $\tilde{\psi}$ is the characteristic function of the jump distribution.

- $\int \tilde{\psi}(\omega e^{-k\,u})\,du$ can be computed explicitly in terms of an exponential integral for double-exponential distribution (Kou model) and numerically using quadrature for Gaussian distribution (Merton model)
Solving the PIDE

- Expand the payoff (assume paid at $t + \Delta t$) in a Fourier basis:

$$
\varphi(x) = v_{t+\Delta t}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega x} F[v_{t+\Delta t}](\omega) d\omega
$$
Solving the PIDE

- Expand the payoff (assume paid at $t + \Delta t$) in a Fourier basis:

$$\varphi(x) = v_{t+\Delta t}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega x} F[v_{t+\Delta t}](\omega) d\omega$$

- Assuming no decisions (such as barrier breach or optimal exercise) are made during the interval $(t, t + \Delta t)$:

$$v_t(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} h_{t,x}(\omega, t + \Delta t) F[v_{t+\Delta t}](\omega) d\omega$$

The above satisfies the PIDE and the boundary condition
Solving the PIDE

- Expand the payoff (assume paid at $t + \Delta t$) in a Fourier basis:

$$\varphi(x) = v_{t+\Delta t}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega x} \mathcal{F}[v_{t+\Delta t}](\omega) \, d\omega$$

- Assuming no decisions (such as barrier breach or optimal exercise) are made during the interval $(t, t + \Delta t)$:

$$v_t(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} h_{t,x}(\omega, t + \Delta t) \mathcal{F}[v_{t+\Delta t}](\omega) \, d\omega$$

The above satisfies the PIDE and the boundary condition.

- Apply Fourier transform to $v_t(x)$:

$$\mathcal{F}[v_t](\omega) = \int_{-\infty}^{\infty} \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\Psi_{t+\Delta t}(\omega') + \Phi_{t+\Delta t}(\omega') x} \mathcal{F}[v_{t+\Delta t}](\omega') \, d\omega' \right] e^{-i\omega x} \, dx$$
Mean-Reverting FST Method

PIDE Solution in Fourier space

\[ \mathcal{F}[\nu_t](\omega) = e^{\Psi_t + \kappa \Delta t}(\omega e^{\kappa \Delta t}) + \kappa \Delta t \mathcal{F}[\nu_{t+\Delta t}](\omega e^{\kappa \Delta t}) \]
Mean-Reverting FST Method

PIDE Solution in Fourier space

\[ \mathcal{F}[\nu_t](\omega) = e^{\Psi_t^t+\Delta t(\omega e^{\kappa \Delta t})+\kappa \Delta t} \mathcal{F}[\nu_{t+\Delta t}](\omega e^{\kappa \Delta t}) \]

- Since \( \kappa > 0, e^{\kappa \Delta t} > 1 \), extrapolation in frequency space of \( \mathcal{F}[\nu_{t+\Delta t}] \) is required.
Mean-Reverting FST Method

**PIDE Solution in Fourier space**

\[
F[v_t](\omega) = e^{\Psi_t^{t+\Delta t}(\omega e^{\kappa \Delta t}) + \kappa \Delta t} F[v_{t+\Delta t}](\omega e^{\kappa \Delta t})
\]

- Since \( \kappa > 0 \), \( e^{\kappa \Delta t} > 1 \), extrapolation in frequency space of \( F[v_{t+\Delta t}] \) is required.

- Using the scaling property of the Fourier transform, this can be obtained by interpolating in real space \( v_{t+\Delta t} \).

**Mean-Reverting FST Method**

\[
v^{m-1}(x) = \text{FFT}^{-1}\left[ e^{\Psi(\omega e^{\kappa \Delta t})} \cdot \text{FFT}[v^m(x e^{-\kappa \Delta t})] \right]
\]

- Without mean reversion, mrFST reduces to the standard FST method of Jackson, Jaimungal and Surkov (2007) since \( \Phi_t^T(\omega) \to i\omega \).
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Spot Price Model

\[ dY_t = \kappa (\theta - Y_{t-}) \, dt + dJ_t \]
\[ X_t = BY_t \]
\[ S_t^i = \exp\{X_t^i\} \quad i = 1, \ldots, n \]

- \( n \) log spot-prices \( X_t \) are modeled as a linear transformation of a set of \( d \)-fundamental market factors \( Y_t \)
- \( \theta \) a \( d \)-dimensional vector of long-run means
- \( \kappa \) a \( d \times d \) matrix with positive eigenvalues representing the mixing of the market factors
- \( B \) a \( d \times n \) matrix representing the linear transformation of the market factors into the observed log-prices
- \( J_t \) a \( d \)-dimensional Lévy process with Lévy triple \((\gamma, C, \nu)\)
Flexible Framework

• One factor mean-reverting model with jumps (*Clewlow and Strickland 2000*):

\[
\theta = \theta, \ \kappa = \kappa, \ C = \sigma^2, \ B = 1, \text{ and } \nu(dZ) = \lambda F(dz)
\]

\[
dX_t = \kappa(\theta - X_t) \, dt + \sigma \, dW_t + dJ_t
\]
Flexible Framework

- One factor mean-reverting model with jumps (Clewlow and Strickland 2000):
  \[ \theta = \theta, \kappa = \kappa, \quad C = \sigma^2, \quad B = 1, \quad \text{and} \quad \nu(dZ) = \lambda F(dz) \]
  \[ dX_t = \kappa(\theta - X_t) \, dt + \sigma \, dW_t + dJ_t \]

- Mean-reverting jump-diffusion model (Hikspoors and Jaimungal 2007) with different decay rates for the jumps and diffusion.
  \[ \theta = \begin{pmatrix} \theta \\ 0 \end{pmatrix}, \quad \kappa = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}, \quad C = \begin{pmatrix} \sigma^2 & 0 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \end{pmatrix} \]
  and \( \nu(dZ_1 \times dZ_2) = \lambda \delta_{Z_1} dF(Z_2) \).
  \[ dY_t^1 = \alpha(\theta - Y_t^1) \, dt + \sigma \, dW_t \]
  \[ dY_t^2 = -\beta Y_t^2 \, dt + dJ_t \]
  \[ X_t = Y_t^1 + Y_t^2 \]
Simulated Electricity Spot Prices
Flexible Framework (cont.)

- Two factor mean-reverting model (Barlow, Gusev, and Lai 2004) with log-prices mean-revert to a stochastic long-run mean, which itself mean-reverts to a fixed level:

\[
\theta = \begin{pmatrix} \theta \\ \theta \end{pmatrix}, \quad \kappa = \begin{pmatrix} \alpha & -\alpha \\ 0 & \beta \end{pmatrix}, \quad C = \begin{pmatrix} \sigma^2 & \rho \sigma \eta \\ \rho \sigma \eta & \eta^2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad \nu(dZ) = 0
\]

\[
dX_t = \alpha(Y_t - X_t)dt + \sigma \, dW^X_t \\
dY_t = \beta(\theta - Y_t)dt + \eta \, dW^Y_t
\]
Two factor mean-reverting model (Barlow, Gusev, and Lai 2004) with log-prices mean-revert to a stochastic long-run mean, which itself mean-reverts to a fixed level:

\[
\begin{pmatrix}
\theta \\
\theta
\end{pmatrix}
\begin{pmatrix}
\alpha & -\alpha \\
0 & \beta
\end{pmatrix}
\begin{pmatrix}
\sigma^2 & \rho \sigma \eta \\
\rho \sigma \eta & \eta^2
\end{pmatrix}
\begin{pmatrix}
1 \\
0
\end{pmatrix}
\nu(dZ) = 0
\]

\[
dX_t = \alpha(Y_t - X_t)dt + \sigma dW_t^X \\
dY_t = \beta(\theta - Y_t)dt + \eta dW_t^Y
\]

Jump diffusion model where the diffusions are correlated, and jumps may have codependent pieces. Noise driven by a copula to introduce co-dependence in the innovations:

\[
\begin{pmatrix}
\theta \\
\phi
\end{pmatrix}
\begin{pmatrix}
\alpha & \gamma \\
\delta & \beta
\end{pmatrix}
\begin{pmatrix}
\sigma^2 & \rho \sigma \eta \\
\rho \sigma \eta & \eta^2
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0 \\
1
\end{pmatrix}
\nu(dZ_1 \times dZ_2) = dC(F_1(Z_1), F_2(Z_2))
\]

with \(C(u, v)\) a copula and \(F_i(z)\) two marginal distribution functions.
Simulated Energy Spot Prices

Graph showing simulated energy spot prices for Oil and Natural Gas over time. The x-axis represents time (t), and the y-axis represents the spot price in dollars ($). The graph illustrates the fluctuating prices of oil and natural gas over a specified period.
Multi-Dimensional Mean-Reverting FST Method

PIDE Solution in Fourier Space

The discounted price \( v_t(Y_t) \) of a European option written on the vector of price processes \( \{ S^1_t = e^{X^1_t}, \ldots, S^n_t = e^{X^n_t} \} \) where \( X_t = B Y_t \) with payoff function \( \varphi(X_T) = \varphi(B Y_T) = \phi(Y_T) \) is

\[
\mathcal{F}[v_t(Y_t)] = e^{\Psi_t(\omega, T) + (T-t)\text{Tr} \kappa} \mathcal{F}[\varphi(Y_t)](e^{\kappa'(T-t)} \omega)
\]

where,

\[
\Psi_t(\omega, T) = \int_t^T \psi(e^{\kappa'(u-t)} \omega) \, du
\]

\[
\psi(\omega) = i \omega' \kappa \theta - \frac{1}{2} \omega' C \omega + \int \left( e^{i \omega' y} - 1 - i \mathbb{1}_{|y| < 1} \omega' y \right) \nu(dy)
\]
Multi-Dimensional Mean-Reverting FST Method

By discretizing space and frequency, prices for a full spectrum of spot values are computed using two FFT evaluations

\[ V_{n-1}(X) = \text{FFT}^{-1} \left[ e^{\Psi_0(\omega, \Delta t) + \Delta t \text{Tr} \kappa} \cdot \text{FFT}[V_n](e^{\kappa' \Delta t} \omega) \right] (X) \]

The transform of the price at time-step \( n \) is required at scaled frequencies. Using the scaling property of Fourier transforms

\[ \mathcal{F} [g] (e^{\kappa' \Delta t} \omega) = \mathcal{F} [\tilde{g}] (\omega) e^{-\Delta t \text{Tr} \kappa} \quad \tilde{g}(X) \triangleq g(X e^{-\kappa' \Delta t}) \]

these are conveniently computed from interpolated prices at time-step \( n \):

\[ V_{n-1}(X) = \text{FFT}^{-1} \left[ e^{\Psi_0(\omega, \Delta t)} \cdot \text{FFT}[	ilde{V}_n](\omega) \right] (X) \]
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## European Option Results

<table>
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<th>N</th>
<th>Value</th>
<th>Change</th>
<th>Convergence order</th>
<th>Time (msec.)</th>
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</table>

- **Option**: European put $S = 100$, $K = 105$, $T = 1$
- **Model**: Merton jump-diffusion with mean reversion
  - $\sigma = 0.2$, $\lambda = 0.25$, $\tilde{\mu} = 0.3$, $\tilde{\sigma} = 0.5$, $\theta = 90$, $\kappa = 0.75$, $r = 0.05$
- **Monte Carlo**: 95% CI $(10.47243025, 10.47356975)$ @ 114 sec.
American Option Results

<table>
<thead>
<tr>
<th>N</th>
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<th>Value</th>
<th>Change</th>
<th>Convergence order</th>
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<td>0.9511</td>
<td>9.546</td>
</tr>
</tbody>
</table>

- **Option**: American put \( S = 100, K = 105, T = 1 \)
- **Model**: Merton jump-diffusion with mean reversion
  \[ \sigma = 0.2, \lambda = 0.25, \bar{\mu} = 0.3, \bar{\sigma} = 0.5, \theta = 90, \kappa = 0.75, r = 0.05 \]
- **Monte Carlo**: 95% CI (12.2330185, 12.2361035) @ 53 min.
### Discrete Barrier Option Results

<table>
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<tr>
<th>N</th>
<th>Value</th>
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</table>

- **Option**: Down-and-out barrier put $S = 100$, $K = 105$, $T = 1$, $B = 85$, $R = 2.5$ with hourly monitoring
- **Model**: Merton jump-diffusion with mean reversion $\sigma = 0.2$, $\lambda = 0.25$, $\bar{\mu} = 0.3$, $\bar{\sigma} = 0.5$, $\theta = 90$, $\kappa = 0.75$, $r = 0.05$
- **Monte Carlo**: 95% CI (3.04777486, 3.05662484) @ 19.1 min.
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## Spread Option Results

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<th>Value</th>
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<th>Time (sec.)</th>
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<tbody>
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</tbody>
</table>

- **Option**: European spread call $S_1 = 100$, $S_2 = 100$, $K = 3.5$, $T = 1$
- **Model**: 2D Kou jump-diffusion with mean reversion and copula
  \[
  \begin{align*}
  \sigma_1 &= 0.2, \lambda_1 = 0.75, \rho_1 = 0.45, \eta_{1+} = 0.25, \eta_{1-} = 0.125, \theta_1 = 92, \kappa_1 = 0.5, \\
  \sigma_2 &= 0.3, \lambda_2 = 0.5, \rho_2 = 0.55, \eta_{2+} = 0.3, \eta_{2-} = 0.2, \theta_2 = 110, \kappa_2 = 0.75 \\
  \rho &= 0.7, r = 0.05 \\
  \lambda_c &= 0.5, \hat{\mu}_{c1} = -0.1, \hat{\sigma}_{c1} = 0.2, \hat{\mu}_{c2} = 0.075, \hat{\sigma}_{c2} = 0.15, \rho_c = 0.7
  \end{align*}
\]
- **Monte Carlo**: 95% CI (20.378096, 20.431361) @ 35 minutes
American Spread Option Results

<table>
<thead>
<tr>
<th>N</th>
<th>M</th>
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<th>Time (sec.)</th>
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<td>2048</td>
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<td>23.48240921</td>
<td>0.0238120</td>
<td>1.0476</td>
<td>353.4</td>
</tr>
</tbody>
</table>

- **Option:** American spread call $S_1 = 100$, $S_2 = 100$, $K = 3.5$, $T = 1$
- **Model:** 2D Kou jump-diffusion with mean reversion and copula
  $\sigma_1 = 0.2$, $\lambda_1 = 0.75$, $p_1 = 0.45$, $\eta_{1+} = 0.25$, $\eta_{1-} = 0.125$, $\theta_1 = 92$, $\kappa_1 = 0.5$, $\sigma_2 = 0.3$, $\lambda_2 = 0.5$, $p_2 = 0.55$, $\eta_{2+} = 0.3$, $\eta_{2-} = 0.2$, $\theta_2 = 110$, $\kappa_2 = 0.75$, $\rho = 0.7$, $r = 0.05$
  $\lambda_c = 0.5$, $\hat{\mu}_c = -0.1$, $\hat{\sigma}_c = 0.2$, $\hat{\mu}_c = 0.075$, $\hat{\sigma}_c = 0.15$, $\rho_c = 0.7$
1 Motivation

2 Mean-Reverting Fourier Space Time-stepping method
   - One-dimensional mean reversion with jump-diffusion
   - Multi-dimensional general framework

3 Numerical Results and Applications
   - Single-asset European, American and barrier options
   - Multi-asset spread options
   - Swing options

4 Conclusions
Overview

- Common in electricity and natural gas markets
- Provides constrained flexibility with respect to volume and timing of energy delivery
- Two components: a pure forward agreement and a swing option

Example: Simple Swing Option

The holder agrees to buy 100MWh at a cost of $45/MWh over a period of 1 month. At the start of each hour, the holder has the right to increase power consumption to 110MW for that hour (swing up) or decrease to 90MW (swing down) at the same cost. The total number of swings is limited to 50.

The swing component is the right to change consumption at holder’s choosing.

For overview of swing options and their valuation see Ware (2007)
At each swing opportunity, a choice to exercise \( q \) swing “options” for immediate cashflow \( \Upsilon \) must be made:

**Dynamic Programming Equation**

\[
\nu_{tm}(x, Q) = \max_q \left\{ \Upsilon_{tm}(x, q) + e^{-r\Delta t}\mathbb{E}[\nu_{tm+1}(x, Q + q)] \right\}
\]

where the expectation is readily computed using the mrFST method.
At each swing opportunity, a choice to exercise $q$ swing “options” for immediate cashflow $\Upsilon$ must be made:

**Dynamic Programming Equation**

$$v_{tm}(x, Q) = \max_q \left\{ \Upsilon_{tm}(x, q) + e^{-r\Delta t} \mathbb{E}[v_{tm+1}(x, Q + q)] \right\}$$

where the expectation is readily computed using the mrFST method.

- The available choices are to do nothing $q = 0$, swing up $q > 0$ or swing down $q < 0$.
- The amount of swings may be bounded $|Q_{tm}| \leq \overline{Q}$ where $Q_{tm} = \sum_{j=1}^{m} q_{tj}$ or $Q_{tm} = \sum |q_{tj}|$.
- The cashflow function $\Upsilon_t(x, q)$ may include a penalty term to enforce additional limits on $Q$ or may be as simple as $\Upsilon_t(x, q) = q(e^x - K)$. 

**Option:** Swing $S = 100$, $K = 100$, $T = 2$, $-3 \leq Q \leq 5$

**Model:** Kou jump-diffusion with mean reversion

$\sigma = 0.3$, $\lambda = 0.7$, $p = 0.45$, $\eta_+ = 0.25$, $\eta_- = 0.2$, $\theta = 90$, $r = 0.05$
1 Motivation

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4 Conclusions
mrFST Method

- Naturally applied to
  - Path-dependent options with discontinuous/irregular payoffs
  - Multi-dimensional problems
  - Exotic options such as swing

- Computationally efficient
  - 2 FFTs required to obtain option values computed for a range of spots
  - European options priced in a single time-step
  - Bermudan style options do not require time-stepping between monitoring dates
Thank You

Questions?