COSTLY MONITORING, LOAN CONTRACTS, AND EQUILIBRIUM CREDIT RATIONING*

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I. INTRODUCTION

The main purpose of this paper is to show that, in a credit market with asymmetrically informed lenders and borrowers and costly monitoring, equilibrium credit rationing of the type discussed by Stiglitz and Weiss [1981] and Keeton [1979] can exist. In equilibrium it may be the case that, among a group of identical would-be borrowers, some receive loans, while others do not. Our model relies on monitoring costs to produce this result, as it contains none of the features that produce rationing in other models. For example, there are no adverse selection or moral hazard problems, as in Stiglitz and Weiss [1981].

An advantage of our approach is that debt contracts can be derived as optimal arrangements between borrowers and lenders; such contracts serve to economize on monitoring costs. The state where a lender monitors a borrower can then be interpreted as bankruptcy, and the monitoring cost as a cost of bankruptcy. Our model is similar in this respect to the costly state verification setups of Townsend [1979] and Gale and Hellwig [1984], though neither of these authors studies the implications of their models for the type of credit rationing examined here. In Stiglitz and Weiss [1981] debt contracts are important in obtaining credit rationing equilibria, but the contract form is imposed exogenously.

Given that the optimal contract is a debt contract, the probability that monitoring occurs and the expected cost of monitoring to the lender increase with the loan interest rate. It may therefore be the case that, in equilibrium, agents who do not receive loans cannot bid loans away from those who do receive them by offering lenders a higher interest rate, since this would decrease the expected return on the loan to the lender.

The equilibrium can be one of two types in our model; either there is credit rationing or there is not. Depending on what type of equilibrium exists, interest rates and the quantity of loans respond quite differently to changes in the environment. For example, there

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are quantity effects in an equilibrium with rationing where these effects are absent in an equilibrium without rationing. This can be interpreted as being consistent with the thrust of the availability doctrine (see Roosa [1951]); monetary policy can have real effects with little or no effect on market interest rates.

The remainder of the paper is organized as follows. In Section II the model is constructed. An examination of optimal contracts and equilibrium is in Section III. In Section IV some comparative statics experiments are discussed. The final section is a summary and conclusion.

II. The Model

In this environment there is a countable infinity of agents indexed by $i = 1, 2, 3, \ldots, \infty$, who each live for two periods. Period 0 is the planning period, and in period 1 consumption takes place. Let d_i denote an agent's type. We can have either $d_i = \ell$ (agent i is a "lender") or $d_i = e$ (agent i is an "entrepreneur").

Each lender receives an indivisible endowment of one unit of an investment good in period 0, which can be lent to an entrepreneur or invested in a project that yields a certain return of t_i units of the consumption good in period one if one unit is invested in period 0, and yields zero units otherwise. Entrepreneurs receive no endowments. However, if $d_i =$ e, then agent i has access to an investment project that produces a random return of $\tilde{w}_i$ units of the consumption in period 1, if the project is funded with one unit of the investment good in period 0, and produces zero units otherwise. The $\tilde{w}_i$'s are independent and identically distributed across entrepreneurs according to the probability density function $f(\cdot)$ and the probability distribution function $F(\cdot)$. The function $f(\cdot)$ is continuously differentiable and positive on $[o, \overline{w}]$, where $\overline{w} > 0$.

The realization of $\tilde{w}_i$, denoted by w_i, is costlessly observable only to agent i, though all agents know $f(\cdot)$. A lender can observe a particular w_i by expending γ units of effort in monitoring, where monitoring decisions are made in period 1. Lenders are each endowed with γ units of effort, and each maximizes the expected value of $u(c,a)$, where c is consumption and a is effort. We assume that

$$u(c,a) = c - a .$$

Entrepreneurs each have an endowment of zero units of effort and maximize the expected value of consumption.

Let $\Omega = \{1, 2, 3, \ldots, \infty\}$. Then, following Billingsley [1979], define a probability measure on the class of subsets of Ω by

$$P_n(A) = (1/n)\#\,[i: 1 \leq i \leq n, i \epsilon A],$$

and let $D(A) = \lim_{n \to \infty} P_n(A)$. Let $h(\cdot)$ denote a probability density function which is positive on $(\underline{t}, \bar{t}]$ and zero otherwise, and let $H(\cdot)$ denote the corresponding distribution function. We have $0 < \underline{t} < \bar{t} < \infty$. Then, the composition of the population is described by

$$D(\{i: d_i = \ell, t_i \leq t'\}) = \alpha H(t')$$

and

$$D(\{i: d_i = e\}) = 1 - \alpha,$$

where $0 < \alpha < 1$. Thus, the fraction of lenders in the population is α. We assume that $\alpha > 1/2$ so that the demand for credit is at least potentially satisfied. Note that lenders face different opportunity returns. The purpose of this device is to generate an upward-sloping supply of funds schedule in the market for credit.

In contrast to Diamond [1984], monitoring decisions are made ex post rather than ex ante. The monitoring technology is a special case of Gale and Hellwig [1985], which considers an environment where monitoring costs are state dependent.

Stiglitz and Weiss [1981] examine two factors that can produce equilibrium credit rationing: adverse selection and moral hazard. Since there is no ex ante informational asymmetry, adverse selection is not a problem in our model. Also, as the actions of entrepreneurs do not affect the returns on investment projects, moral hazard will not be a problem, at least in the ex ante sense of Stiglitz and Weiss. However, as is shown in the next section, ex post moral hazard will be an important factor affecting contractual arrangements between lenders and entrepreneurs.

III. Optimal Contracts and Equilibrium

In period 0, participants in the credit market are, on the demand side, entrepreneurs who offer contracts on the market and, on the supply side, lenders who exchange units of the investment good for these contracts. An interpretation is possible in terms of "banks" that intermediate between lenders and entrepreneurs. However, these banks would be entirely transparent in our analysis. We examine a similar environment where intermediation arises endogenously in Williamson [1986a].

Contracts offered on this market will be evaluated by lenders in terms of the expected return they offer. Expected returns to lenders thus play the role of prices in the credit market. We let r denote the "market expected return" (to be determined endogenously), which all agents treat as a fixed parameter.

We wish to determine the form of the contract which, given r, it is optimal for an entrepreneur to offer on the market in exchange for one unit of the investment good. First, note that contracts must provide for monitoring in some states of the world; otherwise the entrepreneur would maximize consumption by claiming the return on her project to be zero, no matter what the true return is. Therefore, the contract must specify in what states monitoring is to occur, and what the payments to the lender are to be, if monitoring occurs and if it does not.

In period 1, when the borrower observes her return w (note that we drop subscripts here), she emits a signal $w^s \in [0, \overline{w}]$ to the lender. The contract will specify that, if $w^s \in S \subset [0, \overline{w}]$, then monitoring occurs, and if $w^s \notin S$, it does not occur. The payment from the entrepreneur to the lender is $R(w)$ if $w^s \in S$, and $K(w^s)$ if $w^s \notin S$, where $R(\cdot)$ and $K(\cdot)$ are functions on $[0, \overline{w}]$. If $w^s \notin S$, then the entrepreneur will always choose w^s so as to minimize the payment to the lender. Therefore, if $w^s \notin S$, the payment is a constant, denoted by x. It remains to determine the payment schedule $R(w)$, which must satisfy $0 \leq R(w) \leq w$. Incentive compatibility implies that $w^s \in S$ if $R(w) < x$ and $w^s \notin S$ if $R(w) \geq x$.[1] Let $A = \{w: R(w) < x\}$ and $B = \{w: R(w) \geq x\}$. Then, the optimal contract is a payment schedule–"interest rate" pair $\{R(w), x\}$, which maximizes the entrepreneur's expected utility while giving the lender a level of expected utility of at least r:

$$(1) \quad \max_{\{R(w),x\}} \left\{ \int_A [w - R(w)]f(w)dw + \int_B [w - x]f(w)dw \right\}$$

subject to

$$\int_A [R(w) - \gamma]f(w)dw + \int_B x\, f(w)dw \geq r .$$

PROPOSITION. The optimal payment schedule is $R(w) = w$, independent of x.

Proof of the Proposition. Suppose not, and that $[R'(w), x']$ is

1. We assume that if the entrepreneur is indifferent between being monitored and not being monitored, then she chooses the latter.

the optimal contract. First, note that the constraint in (1) must hold with equality, since otherwise we could reduce $R(w)$ for some w such that the constraint would still hold, and increase the value of the objective function. Letting $A' = \{w\colon R'(w) < x'\}$ and $B' = \{w\colon R'(w) \geq x'\}$, we have

$$\int_{A'} [R'(w) - \gamma] f(w) dw + \int_{B'} x' f(w) dw = r .$$

Since $R'(w) < w$ for some $w \in A'$, there exists another payment schedule $R''(w)$ with $R''(w) \geq R'(w)$ for all w and $R''(w) > R'(w)$ for some $w \in A'$, with $R''(w)$ continuous and monotone increasing on $[o, \bar{w}]$. There is then some x'', where $0 < x'' < x'$, such that, with $A'' = \{w\colon x'(w) < x''\}$ and B'' $R = R'$ $\{w\colon x'(w) \geq x''\}$,

$$\int_{A''} [R''(w) - \gamma] f(w) dw + \int_{B''} x'' f(w) dw = r .$$

The change in the objective function in changing the contract from $[R'(w), x']$ to $[R''(w), x'']$ is then

$$\gamma \left[\int_{A'} f(w) dw - \int_{A''} f(w) dw \right] > 0 ,$$

a contradiction.

Q.E.D.

The proposition states that the optimal contract has all the essential features of a debt contract. Either the entrepreneur pays the lender a fixed amount x, in period 1, or the entrepreneur defaults on her debt, monitoring occurs, and the lender receives the entire return on the project. The default state can then be interpreted as bankruptcy and the monitoring cost γ as a cost of bankruptcy. Debt contracts are also derived as optimal contractual arrangements in Gale and Hellwig [1985] and Diamond [1984].[2]

Optimal contracts are completely characterized by the promised payment x. We can then express the expected utility of the contract to the lender and borrower as functions of x. For the lender, expected utility is

$$\text{(2)} \qquad \pi_{\ell}(x) = \int_{o}^{x} w f(w) dw + x[1 - F(x)] - \gamma F(x) ,$$

2. Note that this framework can be trivially extended to include collateral requirements and equity participation by the borrower. Collateral can have a random value in terms of the consumption good that is incorporated in $\tilde{w}_i$ for lender i, with a collateral verification cost which is included in γ. If each project requires y units of the investment good to operate, where $y > 1$, and each entrepreneur is endowed with $y - 1$ units, then "full equity participation" will be optimal, as in Gale and Hellwig [1985].

and for the entrepreneur it is

$$\pi_e(x) = \int_x^{\bar{w}} wf(w)dw - x[1 - F(x)]. \tag{3}$$

Note that the costs of monitoring, equal to $\gamma F(x)$ in expected utility terms, are a net cost to borrower and lender. These costs will be critical to our credit rationing result, since they imply that $\pi_\ell(x)$ is not monotone increasing in x, though $\pi_e(x)$ is monotone decreasing in x. Differentiating equation (2), we get

$$\pi_\ell'(x) = 1 - F(x) - \gamma f(x).$$

Since $f(x) > 0$ for $x \in [o, \bar{w}]$, therefore $\pi_\ell'(\bar{w}) < 0$, so that $\pi_\ell(\bar{x})$ reaches a maximum for $x < \bar{w}$. It is this feature of the payoff function of the lender which, as in Stiglitz and Weiss [1981], can generate equilibrium credit rationing.

To avoid a multiplicity of equilibria and so that a first-order condition is sufficient to characterize a credit rationing equilibrium, we assume that $\pi_\ell(x)$ is concave:

$$f(x) + \gamma f'(x) > 0. \tag{4}$$

DEFINITION. An equilibrium is a loan interest rate x^*, a market expected return r^*, and an aggregate loan quantity q^*, which satisfy

(i) x^* solves $\max_x \pi_e(x)$ subject to $\pi_\ell(x) \geq r^*$

(ii) $q^* = \alpha H(r^*)$

(iii) Either (a) $q^* = 1 - \alpha$ or (*b*) $q^* < 1 - \alpha$ and $\pi_\ell'(x^*) = 0$.

There are then potentially two types of equilibria, those without rationing given by (iiia) in the definition (NRA equilibria), and those with rationing given by (iiib) (RA equilibria). In an RA equilibrium those lenders with $t_i \leq r^*$ lend to entrepreneurs, while those with $t_i > r^*$ invest in their certain return projects. All entrepreneurs offer debt contracts with a promised payment of x^* in period 0, and lenders with $t_i < r^* = \pi_\ell(x^*)$ draw an entrepreneur at random from this group. Entrepreneurs then have a probability of $[q^*/(1 - \alpha)] < 1$ of receiving a loan. Those entrepreneurs who do not receive loans can offer no contract that will bid loans away from those who do receive them or that will draw more lenders into the credit market. This is the case since, given (iiib), x^* is the loan interest rate that maximizes a lender's expected return from a loan contract. Offering to pay a higher x implies a higher probability of default, with larger expected monitoring costs for the lender. This increase in expected monitoring costs exceeds the increase in expected payments to the lender which result from the higher interest rate.

Our model therefore may exhibit equilibrium credit rationing in the sense of Stiglitz and Weiss [1981] and Keeton [1979]. All would-be borrowers are identical, ex ante, but some may receive loans, while others do not. This type of rationing is to be contrasted with that in Jaffee and Russell [1976] and Gale and Hellwig [1985], where agents are rationed in the sense that they cannot borrow all they would like given the quoted interest rate. Note that if we were to change our model to allow for heterogeneous borrowers, then we may not get credit rationing as Stiglitz and Weiss define it. For example, as in Williamson [1986b], we could allow entrepreneurs' monitoring costs to vary over a continuum. Then, in equilibrium, there would be some cutoff level γ^*, such that entrepreneurs with $\gamma \leq \gamma^*$ receive loans and those with $\gamma > \gamma^*$ do not. However, agents who do not receive loans are "rationed" in this case, in the sense that they would be willing to pay interest rates higher than market rates to receive loans, but no one would be willing to lend to them at any interest rate.

IV. Comparative Statics

One of the attractive features of this model is that comparative statics results are quite easy to obtain. In an NRA equilibrium, q, x, and r are determined by the following three equations, from the definition in the previous section, and equation (2):

$$\int_0^x w f(w)dw + x[1 - F(x)] - \gamma F(x) = r \tag{5}$$

$$q = \alpha H(r) \tag{6}$$

$$q = 1 - \alpha . \tag{7}$$

Equation (5) follows from the optimal choice of a contract offer for each entrepreneur, given r. Since $\pi_e'(x) < 0$, each entrepreneur will offer to pay an interest rate such that the lender receives no more than the market expected return on the contract.

In the case of an RA equilibrium, q, x, and r are determined by (5), (6), and

$$1 - F(x) - \gamma f(x) = 0, \tag{8}$$

where equation (8) is the condition that $\pi_\ell'(x) = 0$; i.e. the loan interest rate maximizes the expected return to the lender.

Let r^* and x^* be the values of r and x, respectively, which solve (5) and (8). Then, if $r^* \geq \underline{t}$, an equilibrium exists, and it is unique. If $\alpha\, \mathrm{H}\,(r^*) > 1 - \alpha$, then the equilibrium is NRA; otherwise it is RA.

We look at three different comparative statics experiments; (1) an increase in each lender's opportunity return; (2) a change in the monitoring cost γ; (3) a mean-preserving spread in the distribution of project returns.

For the first experiment, which involves a shift in the function $H(\cdot)$, the results are straightforward. In the NRA equilibrium, q is determined by (7), so that from (6), the experiment can be characterized as a differential increase in r. Since $\pi_\ell'(x) > 0$ (ignoring the borderline case) in equilibrium, therefore x must increase. In the RA equilibrium, x and r are determined by (5) and (8), so that neither variable is affected. From (6) the only effect is a decrease in q.

Thus, in the NRA equilibrium, interest rates increase, and there are no quantity effects, while in the RA equilibrium there are quantity effects, but no change in interest rates. This might be viewed as being consistent with the thrust of the availability doctrine; monetary policy can have real effects without changing interest rates significantly (see Roosa [1951]). In this model, monetary policy would have to affect *some* real return(s) (i.e., the t_i's) in order to have this effect. It may appear that this is not consistent with the availability doctrine, since the interest rate which need not change (much), according to this doctrine, is usually thought to be the interest rate on the securities in which the central bank conducts its open market operations. Such an interest rate would be incorporated in our model in the schedule of t_i's.

However, consider a model where our credit market submodel is embedded in a more fully specified dynamic general equilibrium framework. Lenders may hold one-period government bonds and make loans to entrepreneurs, and agents other than lenders hold the stock of currency. The government finances its deficit by printing currency and issuing bonds. Then, if there is credit rationing in equilibrium, the real rate of interest on bonds is essentially fixed at the margin, and a permanent increase in the ratio of bonds to currency, interpreted as an open market sale, would lead to a crowding out of lending in the credit market with no effect on the real rate of interest on bonds. Working out the details of this model is outside the scope of this paper. However, see Williamson [1986b] for a dynamic general equilibrium model which incorporates some of the features of our credit market model.

For the NRA equilibrium, a change in γ has no effect on q or r, and the effect on x can be obtained by totally differentiating equation (5) to get

$$\frac{dx}{d\gamma} = \frac{F(x)}{1 - F(x) - \gamma f(x)} > 0. \tag{9}$$

For the RA equilibrium, totally differentiate (5) and (8) and solve to get

$$\frac{dx}{d\gamma} = -\frac{f(x)}{f(x) + \gamma f'(x)} < 0 \tag{10}$$

$$\frac{dr}{d\gamma} = -F(x) < 0. \tag{11}$$

Given (11) and (6), q falls with an increase in γ.

We therefore get quite different effects depending on whether the equilibrium is NRA or RA. In the first case the loan interest rate increases, and in the latter it decreases. Equation (9) is the qualitative effect we would expect; an increase in costs causes the "product price" to increase in a competitive market. The reason we get (10) is that, from (8), an increase in γ causes a decrease in the loan interest rate at which the expected return to the lender is maximized.

In considering the effect of meaning-preserving spreads in equilibrium, it is useful to rewrite equation (2) as

$$\pi_{\ell}(x) = x - \int_0^x F(w)dw - \gamma F(x). \tag{12}$$

The negative of the second and third terms on the right-hand side of (11) can then be interpreted as a risk premium. With $\gamma = 0$, mean-preserving spreads in the project return distribution which increase risk in the sense of Rothschild and Stiglitz [1970] will result in an increase in the risk premium.[3] However, with $\gamma > 0$, mean-preserving spreads have an indeterminate effect on $\gamma F(x)$ (the expected cost of monitoring), and the net effect on the risk premium is therefore indeterminate. Since the comparative statics implications of mean-preserving spreads are determined by the resulting effect on the risk premium, results will depend on the type of mean-preserving spread we consider.

Suppose that we consider a mean-preserving spread in the

3. If we have an adverse selection environment where risk varies across projects, then if $\gamma = 0$, the expected payoff to the lender decreases with risk as in Stiglitz and Weiss [1981].

project return distribution about x^*, the equilibrium level of x. That is, we change $f(w)$ to $f^*(w) = f(w) + \delta g(w)$, where $0 < \delta \leq 1$. Let $G(w) = \int_0^w g(t)dt$. We assume that $G(x^*) = 0, f(w) + g(w) > 0$ for $w \in [o, \overline{w}]$, $\int_0^{\overline{w}} zg(z)dz = 0$, and $\int_0^w G(z)dz \geq 0$ for $w \in [o, \overline{w}]$. For the NRA equilibrium, substituting in equation (5), totally differentiating, and setting $\delta = 0$, we obtain

$$\left.\frac{dx}{d\delta}\right|_{\delta=0} = \frac{\int_0^x w\, g(w)dw}{1 - F(x) - \gamma f(x)} > 0.$$

For the RA equilibrium, following a similar procedure,

$$\left.\frac{dx}{d\delta}\right|_{\delta=0} = -\frac{\gamma g(x)}{f(x) + \gamma f'(x)} > 0$$

$$\left.\frac{dr}{d\delta}\right|_{\delta=0} = \int_0^x wg(w)dw < 0.$$

The results are qualitatively similar in both types of equilibria. We observe a type of risk premium effect where, due to the nature of the debt contract and because of costly bankruptcy, the loan interest rate increases as project risk increases. Note that, in either equilibrium, the size of the increase in x is larger for larger γ.

Conclusion

In this paper we have constructed a credit market model with asymmetric information and costly monitoring. This model can exhibit equilibrium credit rationing, in spite of the absence of adverse selection and moral hazard. Debt contracts were derived as optimal arrangements between lenders and borrowers. Some comparative statics experiments showed that the model responds quite differently to changes in underlying parameters, depending on whether or not there is credit rationing in equilibrium. Quantitative responses are always different, and qualitative responses sometimes are as well.

The model is attractive due to its analytical tractability, and to its usefulness in studying richer phenomena. For example, more complex models with some of the same features are used to study financial intermediation in Williamson [1986a] and business cycle behavior in Williamson [1986b].

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