Expectations and the Neutrality of Money

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Objective and motivation

- Philips curve
- Monetary business cycles
Source of data: Economic Report of the President, 1985 pages 239, 266.
Non-neutrality of money

• Standard Neo-classical model: Money is neutral

\[
\frac{M}{Y} = P
\]
How to Model Non-neutrality of money? — all forms of "money illusion" are rigorously excluded

Difficulties:
- All prices are market clearing
- All agents behave optimally in light of their objectives and expectations
- Expectations are formed optimally

Neoclassical with incomplete information
- Phelps (1969) - Philips curve
- Lucas - Monetary business cycle
How to Model Non-neutrality of money? — real and nominal shocks

- **Two shocks**
  - Real shock
    - Exchange takes place in two physically separated markets
    - Allocation of traders stochastic
  - Money supply is stochastic

- **One signal and two unknown shocks**
  - Traders observe prices in their own market but not the other market
  - An increase in price could be caused by increased money supply or increased demand (number of buyers)
Environment

- Overlapping generations model - live for two periods
  - Each period, $N$ identical individuals are born
  - Population $2N$, $N$ of age 0, $N$ of age 1
  - Supply of labor: each person provides $n$ during the first period of life
  - Production: $n$ units of labor generate $n$ units of output
  - $c^0 + c^1 \leq n, c^0, c^1, n \geq 0$
- Money could be given to each old at the beginning of each period
- Quantity of transfer: proportional to the pretransfer holdings of each
- No inheritance
Exchange and allocation of traders

- **Exchange:**
  - Young supplies consumption goods
  - Old has money

- **Two separate markets:**
  - Old people are equally allocated across the two markets
  - New borns are stochastic
    - one $\theta/2$, the other $1 - \theta/2$
Money and information

- Information of money supply
  - Pretransfer money: known to all agents, \( m \)
  - Posttransfer balances: \( m' \)
    - old knows
    - young does not know exactly, conjecture from price signal

- Evolution of money

\[
m' = xm
\]

\( x \) independent across time, identical in different periods, continuous density function \( f \) on \((0, \infty)\)
Allocation variable \( \theta \) is unknown, conjecture from price signal
\( \theta \) is i.i.d with \( g \) on \((0, 2)\)
Preferences

- **Old:**
  
  Spend all the money inelastically

**Young**

\[ U(c, n) + E \{ V(c') \} \]

Assume

\[ U_{cn} + U_{nn} < 0, \; U_{cc} + U_{cn} < 0 \]

\[
\frac{c'V''(c')}{V'(c')} \leq -a < 0
\]

\[
\lim_{c' \to 0} V'(c') = \infty, \quad \lim_{c' \to \infty} V'(c') = 0
\]
Optimization problem

\[
\max_{c,n,\lambda \geq 0} \left\{ U(c, n) + \int V \left( \frac{x' \lambda}{p'} \right) dF(x', p' | m, p) \right\}
\]

Budget constraints:

\[
p c + \lambda = p n
\]

\[
c' = \frac{x' \lambda}{p'}
\]

\(\lambda\) money from sale

\(F(x', p' | m, p)\) : distribution of \((x', p')\) conditional on \(m,\) and \(p\)
Solution

- Kuhn-Tucher conditions

\[ U_c(c, n) - p\mu \leq 0, \text{ with equality if } c > 0 \]
\[ U_n(c, n) + p\mu \leq 0, \text{ with equality if } n > 0 \]
\[ p(n - c) - \lambda \leq 0, \text{ with equality if } \mu > 0 \]
\[ \int V' \left( \frac{x'\lambda}{p'} \right) \frac{x'}{p'} dF(x', p'|m, p) - \mu \leq 0, \text{ with equality if } \lambda > 0 \]
Solution

- Kuhn-Tucher conditions

  \[ U_c(c, n) - p\mu \leq 0, \text{ with equality if } c > 0 \]
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- Equilibrium money:

  \[ \frac{mx}{\theta} = \lambda \]
Solution

- Kuhn-Tucker conditions

\[ U_c(c, n) - p\mu \leq 0, \text{ with equality if } c > 0 \]

\[ U_n(c, n) + p\mu \leq 0, \text{ with equality if } n > 0 \]

\[ p(n - c) - \lambda \leq 0, \text{ with equality if } \mu > 0 \]

\[ \int V' \left( \frac{x'\lambda}{p'} \right) \frac{x'}{p'} dF(x', p'|m, p) - \mu \leq 0, \text{ with equality if } \lambda > 0 \]

- Equilibrium money:

\[ \frac{mx}{\theta} = \lambda \]

- Function for price

\[ h\left( \frac{\lambda}{p} \right) \frac{1}{p} = \int V' \left( \frac{x'\lambda}{p'} \right) \frac{x'}{p'} dF(x', p'|m, p) \]
Conclusion

- Prove the existence of the equilibrium
- Characterize price function: increases in $x$
- Money’s short-run real effect